King’s College London

TOMSY Project Report on

The Kinematics of KCL Five Fingered Metamorphic Hand

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1 Description

The KCL five fingered metamorphic hand, shown in Fig. 1, comprises of a metamorphic palm and five fingers. Each finger is very simple, just some links connected by revolute joints whose axes are parallel. The interesting part is the palm. It is a spherical five-bar linkage. It is made out of five links in a circular configuration with every joint axis $z_i$ passing through the centre of the sphere, as shown in Fig. 2.

Of the five joints $\theta_i$ on the palm, only $\theta_1$ and $\theta_5$ are actuated. The remaining joints, $\theta_2$, $\theta_3$ and $\theta_4$ are rotating freely, based on the constraints imposed by the geometry of the spherical linkage. Each finger is actuated by only one tendon and move on it’s own finger operation plane, as shown in Fig. 3.

2 Palm Kinematics

The coordinates of the points $A$, $B$, $C$, $D$ and $E$ as well as joint angles $\theta_2$, $\theta_3$ and $\theta_4$ should be computed first. We start by assuming that point $E$ is $p_E = [0, 0, 1]$. Then the coordinates of points $A$, $B$ and $D$ are computed from the known and actuated joint angles $\theta_5$ and $\theta_1$. Then, angle $\theta_3$ can be computed by applying the cosine law for spherical triangles on the triangle $\triangle BCD$. Computing angle $\theta_4$ can be done by adding together angles $\angle EDB$, 

Figure 1: Five Fingered Metamorphic Hand.
Figure 2: Metamorphic Palm and Finger Attachment Points.

\[ \angle BDC \] and subtracting \( \pi \). Angle \( \theta_3 \) can be computed in a similar way, by adding \( \angle ABD, \angle DBC \) and subtracting \( \pi \). This indicates that the distance \( \| BD \| \) has to be computed.

### 2.1 Angles \( \alpha_1 \) to \( \alpha_5 \)

The values for the angles \( \alpha_1 \) to \( \alpha_5 \) are as follows:

\[
\begin{align*}
\alpha_1 &= 25^\circ \\
\alpha_2 &= 40^\circ \\
\alpha_3 &= 70^\circ \\
\alpha_4 &= 112^\circ \\
\alpha_5 &= 113^\circ
\end{align*}
\]

### 2.2 Points \( A, B, D, E \)

The coordinates for points \( A, B, D \) and \( E \) can be computed by performing the rotations described in equations 6, 7, 8 and 9.
Figure 3: Finger Operational Planes.

\[ p_A = R(y_5, \alpha_5) \mathbf{k} \]  \hspace{1cm} (6)
\[ p_B = R(y_5, \alpha_5) R(z_1, \theta_1) \mathbf{k} \]  \hspace{1cm} (7)
\[ p_D = R(z_5, -\theta_5) R(y_4, -\alpha_4) \mathbf{k} \]  \hspace{1cm} (8)
\[ p_E = \begin{bmatrix} 0 \\ 0 \\ R \end{bmatrix} \]  \hspace{1cm} (9)

2.3 Joint Angles \( \theta_2, \theta_3, \theta_4 \)

First, the distance \( \|BD\| \) and the angle \( \alpha_{bd} \) are computed.
\[ \mathbf{bd} = \mathbf{p}_b - \mathbf{p}_d \quad (10) \]
\[ \|BD\| = \sqrt{\mathbf{bd}' \mathbf{bd}} \quad (11) \]
\[ \alpha_{BD} = \arccos \left( 1 - \frac{\|BD\|^2}{2} \right) \quad (12) \]

Then, \( \theta_3 \) is computed by the spherical law of cosines.

\[ \angle BCD = \arccos \left( \frac{\cos \alpha_{BD} - \cos \alpha_2 \cos \alpha_3}{\sin \alpha_2 \sin \alpha_3} \right) \quad (13) \]
\[ \theta_4 = \angle BCD - \pi \quad (14) \]

Next, \( \alpha_{BE} \) is computed and then used to compute \( \theta_4 \).

\[ \mathbf{be} = \mathbf{p}_b - \mathbf{p}_e \quad (15) \]
\[ \|BE\| = \sqrt{\mathbf{be}' \mathbf{be}} \quad (16) \]
\[ \alpha_{BE} = \arccos \left( 1 - \frac{\|BE\|^2}{2} \right) \quad (17) \]

By using \( \alpha_{BE} \), \( \theta_4 \) is computed.

\[ \angle EDB = \arccos \left( \frac{\cos \alpha_{BE} - \cos \alpha_4 \cos \alpha_{BD}}{\sin \alpha_4 \sin \alpha_{BD}} \right) \quad (18) \]
\[ \angle BDC = \arccos \left( \frac{\cos \alpha_2 - \cos \alpha_3 \cos \alpha_{BD}}{\sin \alpha_3 \sin \alpha_{BD}} \right) \quad (19) \]
\[ \theta_4 = (\angle EDB + \angle BDC) - \pi \quad (20) \]

Finally, \( \alpha_{AD} \) is computed and then used to compute \( \theta_3 \). It must be noted that the angles of a spherical triangle don’t add up to \( \pi \). Because of this fact, angle \( \theta_3 \) has to be computed in the same fashion as angles \( \theta_2 \) and \( \theta_4 \).

\[ \mathbf{ad} = \mathbf{p}_a - \mathbf{p}_d \quad (21) \]
\[ \|AD\| = \sqrt{\mathbf{ad}' \mathbf{ad}} \quad (22) \]
\[ \alpha_{AD} = \arccos \left( 1 - \frac{\|AD\|^2}{2} \right) \quad (23) \]
By using $\alpha_{AD}$, $\theta_4$ is computed.

\[
\begin{align*}
\angle ABD & = \arccos \left( \frac{\cos \alpha_{AD} - \cos \alpha_1 \cos \alpha_{BD}}{\sin \alpha_1 \sin \alpha_{BD}} \right) \tag{24} \\
\angle DBC & = \arccos \left( \frac{\cos \alpha_3 - \cos \alpha_2 \cos \alpha_{BD}}{\sin \alpha_2 \sin \alpha_{BD}} \right) \tag{25} \\
\theta_2 & = \pi - (\angle ABD + \angle DBC) \tag{26}
\end{align*}
\]

### 2.4 Point C

Point $C$ can now be located in two different ways. One way is to move in a clockwise direction ($E$ to $D$ to $C$) and the other way is to move in a counter clockwise direction ($E$ to $A$ to $B$ to $C$). The first way is chosen since it is the least computationally intensive. One can use both ways and see if the equations produce the same coordinates in order to verify their correctness.

\[
\mathbf{p}_C = R(z_5, -\theta_5) \ R(y_4, -\alpha_4) \ R(z_4, -\theta_4) \ R(y_3, -\alpha_3) \ \mathbf{k} \tag{27}
\]

### 3 MCP Joints

After determining angles $\theta_2$, $\theta_3$ and $\theta_4$ from equations 26, 14 and 20, the points the fingers attach to the palm, as depicted in Fig. 4 have to be determined.

Angles $\delta_1$ to $\delta_5$ determine points $F_1$ to $F_5$ where each MCP joint attaches on the palm and angles $\gamma_2$ to $\gamma_5$ are determined and fixed so that each finger, except for the thumb, is parallel and co-linear with the arm. $\gamma_1$ is 0.
3.1 **Angles \( \delta_1 \) to \( \delta_5 \)**

The values of the angles \( \delta_1 \) to \( \delta_5 \) are given by equations 28, 29, 30, 31 and 32.

\[
\begin{align*}
\delta_1 &= \alpha_2 \frac{1}{2} \quad (28) \\
\delta_2 &= \alpha_3 \frac{1}{4} \quad (29) \\
\delta_3 &= \alpha_4 \frac{1}{7} \quad (30) \\
\delta_4 &= \alpha_4 \frac{3}{7} \quad (31) \\
\delta_5 &= \alpha_4 \frac{5}{7} \quad (32)
\end{align*}
\]
3.2 Angles $\gamma_1$ to $\gamma_5$

The values of the angles $\gamma_1$ to $\gamma_5$ are given by equations 34, 34, 35, 36 and 37.

\begin{align*}
\gamma_1 &= 0 \quad (33) \\
\gamma_2 &= -\delta_2 - (\alpha_4 - \frac{\pi}{2}) \quad (34) \\
\gamma_3 &= \delta_3 - (\alpha_4 - \frac{\pi}{2}) \quad (35) \\
\gamma_4 &= \delta_4 - (\alpha_4 - \frac{\pi}{2}) \quad (36) \\
\gamma_5 &= \delta_5 - (\alpha_4 - \frac{\pi}{2}) \quad (37)
\end{align*}

3.3 Points $F_i$ and $M_i$

In order to obtain the coordinates that describe the MCP joints, the homogeneous transform matrices to points $F_i$ need to be determined.

\begin{align*}
R_{F_1} &= R(y_5, \alpha_5) \ R(z_1, \theta_1) \ R(y_1, \alpha_1) \ R(z_2, \theta_2) \ R(y_2, \delta_1) \quad (38) \\
R_{F_2} &= R(z_5, -\theta_3) \ R(y_4, -\alpha_4) \ R(z_4, -\theta_4) \ R(y_3, \delta_2) \quad (39) \\
R_{F_3} &= R(z_5, -\theta_3) \ R(y_4, -(\alpha_4 - \delta_3)) \quad (40) \\
R_{F_4} &= R(z_5, -\theta_3) \ R(y_4, -(\alpha_4 - \delta_4)) \quad (41) \\
R_{F_5} &= R(z_5, -\theta_3) \ R(y_4, -(\alpha_4 - \delta_5)) \quad (42)
\end{align*}

After obtaining the rotation matrices $R_{F_i}$, the coordinates of points $F_i$ are easily obtained by eq. 43.

\begin{equation}
f_i = R_{F_i} \ k \quad (43)
\end{equation}

Then, the homogeneous transform matrices for points $F_i$ are given by eq. 44.

\begin{equation}
D_{F_i} = \begin{bmatrix} R_{F_i} & R_{F_i} \ k \\ 0 & 1 \end{bmatrix} \quad (44)
\end{equation}

Next, matrices $D_{F_i}$ are multiplied by the homogeneous transform matrix $D_{FM_i}$ given by eq. 45.
The homogeneous transform matrices to points $M_1$ to $M_5$ are given by eq. 46 and eq. 47.

\[
D_{FM_i} = \begin{bmatrix}
\cos \gamma_i & 0 & -\sin \gamma_i & -a_{i0} \sin \gamma_i \\
0 & 1 & 0 & 0 \\
\sin \gamma_i & 0 & \cos \gamma_i & a_{i0} \cos \gamma_i \\
0 & 0 & 0 & 1
\end{bmatrix}
\] (45)

The coordinates for the points $M_1$ to $M_5$ can be computed from eq. 49.

\[
\begin{bmatrix}
\mathbf{r}_{i0} \\
0
\end{bmatrix} = D_{M_i} \begin{bmatrix}
0 \\
0 \\
0 \\
1
\end{bmatrix}
\] (49)
References


Appendices

Appendix A  MATLAB® Code

```matlab
hold off
clear all
clc
syms R real
syms alpha1 alpha2 alpha3 alpha4 alpha5 real
syms theta1 theta2 theta3 theta4 theta5 real
syms a11 a12 a13 a21 a22 a23 a31 a32 a33 real
syms theta11 theta12 theta13 theta21 theta22 theta23 ...
    theta31 theta32 theta33 real
syms g1 g2 g3 g4 g5 real
syms a10 a20 a30 a40 a50 real
syms d1 d2 d3 d4 d5 real
%% Palm Geometry
q = pi/180;
%% Palm radius
R = 1;

%% Palm input angles
theta1 = 10 * q;
theta5 = 10 * q;

%% Palm link angles
alpha1 = 25 * q;
alpha2 = 40 * q;
alpha3 = 70 * q;
alpha4 = 112 * q;
alpha5 = 113 * q;

%% Finger Geometry
% MCP joint length
a10 = 0.1;
a20 = 0.1;
a30 = 0.1;
a40 = 0.1;
a50 = 0.1;
% MCP joint on palm vector angle
d1 = alpha2/2;
```
d2 = alpha3*1/4;
d3 = alpha4*1/7;
d4 = alpha4*3/7;
d5 = alpha4*5/7;

% MCP joint to palm angle
g1 = 0;
g2 = -d2 - alpha4 + 90*q;
g3 = d3 - (alpha4 - 90*q);
g4 = d4 - (alpha4 - 90*q);
g5 = d5 - (alpha4 - 90*q);

% Finger link length
a11 = 0.5;
a12 = 0.5;
a13 = 0.5;
a21 = 0.5;
a22 = 0.5;
a23 = 0.5;
a31 = 0.5;
a32 = 0.5;
a33 = 0.5;

% Finger angle
theta11 = 0;
theta12 = 0;
theta13 = 0;
theta21 = 0;
theta22 = 0;
theta23 = 0;
theta31 = 0;
theta32 = 0;
theta33 = 0;

% Palm Joint Coordinates (Points A, B, D and E)
% Z-axis Unit Vector
z = [0; 0; 1];
% Z-axis Vector including palm radius.
k = [0; 0; R];

% Rotation matrices, counter clockwise
[−, RY1.ccw, RZ1.ccw] = Rotation(0, alpha1, theta1);
[−, RY5.ccw, −] = Rotation(0, alpha5, 0);
% Rotation matrices, clockwise
[−, RY4 cw, −] = Rotation(0, −alpha4, 0);
\[-, \text{RY}_3\text{cw}, \neg\] = Rotation(0, -alpha3, 0);
\[
\neg, \neg, \text{RZ}_5\text{cw}\] = Rotation(0, 0, -theta5);

% Palm joint coordinates
pa_{ccw} = \text{RY}_5\text{ccw} * k;
pb_{ccw} = \text{RY}_5\text{ccw} * \text{RZ}_1\text{ccw} * \text{RY}_1\text{ccw} * k;
\text{pd}_{cw} = \text{RZ}_5\text{cw} * \text{RY}_4\text{cw} * k;
%pd\text{ccw} = \text{RY}_5\text{ccw} * \text{RZ}_1\text{ccw} * \text{RY}_1\text{ccw} * \text{RZ}_2\text{ccw} * \text{RY}_2\text{ccw} ...
\text{* RZ}_3\text{ccw} * \text{RY}_3\text{ccw} * k;
pe = k;

%%% Joint Angles th2 and th4
ca1 = \cos(alpha1);
ca2 = \cos(alpha2);
ca3 = \cos(alpha3);
ca4 = \cos(alpha4);
ca5 = \cos(alpha5);
sa1 = \sin(alpha1);
sa2 = \sin(alpha2);
sa3 = \sin(alpha3);
sa4 = \sin(alpha4);
sa5 = \sin(alpha5);

BD_{v} = pb_{ccw} - pd_{cw};
BD = \sqrt{BD_{v}' * BD_{v}};
cBD = 1 - BD^2 / 2;
bD = \acos(cBD);
SBD = \sin(bD);

cbd = \acos((cBD - ca2*ca3) / (sa2 * sa3));
theta3 = cbd - pi;

BE_{v} = pb_{ccw} - pe;
BE = \sqrt{BE_{v}' * BE_{v}};
cBE = 1 - BE^2 / 2;
bE = \acos(cBE);
SBE = \sin(bE);
edb = \acos((cBE - ca4*cBD) / (sa4 * sBD));
bdc = \acos((ca2 - ca3*cBD) / (sa3 * sBD));
theta4 = (edb + bdc) - pi;

%
% Theta2 needs some work
AD_v = pa_ccw - pd_bw;
AD = sqrt((AD_v') * AD_v);
cAD = 1 - AD^2 / 2;
ad = acos(cAD);
sAD = sin(ad);

abd = acos((cAD - calCBD) / (sa1*sBD));
dbc = acos((ca3 - ca2*BD) / (sa2*sBD));
theta2 = -(abd + dbc) - pi;

% calc cont
[¬, RY2_ccw, RZ2_ccw] = Rotation(0, alpha2, theta2);
[¬, ¬, RZ4_cw]=Rotation(0, 0, theta4);

pc_ccw = RY5_ccw * RZ1_ccw * RY1_ccw * RZ2_ccw * RY2_ccw * k;

% verify th4 and th2
[¬, RY2_cw, RZ3_cw] = Rotation(0, -alpha2, theta3);

pb_cw = RZ5_cw * RY4_cw * RZ4_cw * RZ3_cw * ... 
       RZ2_cw * k;

% verify th4 and th2
[¬, RY3_cw, RZ3_cw] = Rotation(0, alpha3, -theta3);

% Finger Origins (Points Mi)
% Rotation matrices for location on palm
[¬, RY4f5_cw, ¬] = Rotation(0, -(alpha4-d5), 0);
[¬, RY4f4_cw, ¬] = Rotation(0, -(alpha4-d4), 0);
[¬, RY4f3_cw, ¬] = Rotation(0, -(alpha4-d3), 0);
[¬, RY3f2_cw, ¬] = Rotation(0, -d2, 0);
[¬, RY2f1_cw, ¬] = Rotation(0, d1, 0);

% finger #1 (Thumb)
RF1 = RY5_cw * RZ1_cw * RY1_cw * RZ2_cw * RY2f1_cw;
DF1 = [RF1 RF1*k; 0 0 0 1];
DFM1 = [cos(g1) 0 -sin(g1) -a10*sin(g1);...
      0 1 0 0;...]
\[
\begin{align*}
\sin(g_1) & \ 0 \ \cos(g_1) \ a_{10} \cos(g_1); \\
0 & \ 0 \ 0 \ 1; \\
\text{DM1} & = \text{DF1} \cdot \text{DFM1}; \\
r_{10} & = \text{DM1} \cdot [0;0;0;1]; \\
r_{10} & = r_{10}(1:3,1); \\
r_{11} & = \text{RF1} \cdot k; \\
\% \ \text{finger #2 (Index)} \\
\text{RF2} & = \text{RZ5}_{cw} \ast \text{RY4}_{cw} \ast \text{RZ4}_{cw} \ast \text{RY3f2}_{cw}; \\
\text{DF2} & = [\text{RF2} \ \text{RF2} \cdot k; \ 0 \ 0 \ 0 \ 1]; \\
\text{DFM2} & = [\cos(g_2) \ 0 \ -\sin(g_2) \ -a_{20} \sin(g_2); \\
& \quad 0 \ 1 \ 0 \ 0; \\
& \quad \sin(g_2) \ 0 \ \cos(g_2) \ a_{20} \cos(g_2); \\
& \quad 0 \ 0 \ 0 \ 1]; \\
\text{DM2} & = \text{DF2} \cdot \text{DFM2}; \\
r_{20} & = \text{DM2} \cdot [0;0;0;1]; \\
r_{20} & = r_{20}(1:3,1); \\
r_{21} & = \text{RF2} \cdot k; \\
\% \ \text{finger #3} \\
\text{RF3} & = \text{RZ5}_{cw} \ast \text{RY4}_{f3}_{cw}; \\
\text{DF3} & = [\text{RF3} \ \text{RF3} \cdot k; \ 0 \ 0 \ 0 \ 1]; \\
\text{DFM3} & = [\cos(g_3) \ 0 \ -\sin(g_3) \ -a_{30} \sin(g_3); \\
& \quad 0 \ 1 \ 0 \ 0; \\
& \quad \sin(g_3) \ 0 \ \cos(g_3) \ a_{30} \cos(g_3); \\
& \quad 0 \ 0 \ 0 \ 1]; \\
\text{DM3} & = \text{DF3} \cdot \text{DFM3}; \\
r_{30} & = \text{DM3} \cdot [0;0;0;1]; \\
r_{30} & = r_{30}(1:3,1); \\
r_{31} & = \text{RF3} \cdot k; \\
\% \ \text{finger #4} \\
\text{RF4} & = \text{RZ5}_{cw} \ast \text{RY4}_{f4}_{cw}; \\
\text{DF4} & = [\text{RF4} \ \text{RF4} \cdot k; \ 0 \ 0 \ 0 \ 1]; \\
\text{DFM4} & = [\cos(g_4) \ 0 \ -\sin(g_4) \ -a_{40} \sin(g_4); \\
& \quad 0 \ 1 \ 0 \ 0; \\
& \quad \sin(g_4) \ 0 \ \cos(g_4) \ a_{40} \cos(g_4); \\
& \quad 0 \ 0 \ 0 \ 1]; \\
\text{DM4} & = \text{DF4} \cdot \text{DFM4}; \\
r_{40} & = \text{DM4} \cdot [0;0;0;1]; \\
r_{40} & = r_{40}(1:3,1); \\
r_{41} & = \text{RF4} \cdot k; \\
\% \ \text{finger #5} \\
\text{RF5} & = \text{RZ5}_{cw} \ast \text{RY4f5}_{cw};
\end{align*}
\]
DF5 = [RF5 RF5*k; 0 0 0 1];
DFM5 = [cos(g5) 0 −sin(g5) −a50*sin(g5);...
0 1 0 0;...
−sin(g5) 0 cos(g5) a50*cos(g5);...
0 0 0 1];
DM5 = DF5*DFM5;
r50 = DM5*[0;0;0;1];
r50 = r50(1:3,1);
r51 = RF5*k;

%% Fingertip Coordinates (Points Ti)
% Transformation from finger base to finger tip
[RX11,RY11,RZ11]=Rotation(theta11,0,0);
[RX12,RY12,RZ12]=Rotation(theta12,0,0);
[RX13,RY13,RZ13]=Rotation(theta13,0,0);
T11=[RX11, RX11*[0; 0; a11]; 0,0,0,1];
T12=[RX12, RX12*[0; 0; a12];0,0,0,1];
T13=[RX13, RX13*[0; 0; a13];0,0,0,1];
T1=T11*T12*T13;

%% plots
palm = [pa_ccw'; pb_ccw'; pc_cw'; pd_cw'; pe'];
fingers = [r10'; r20'; r30'; r40'; r50'];
mcp = [r11'; r21'; r31'; r41'; r51'];
plot3(palm(:,1),palm(:,2),palm(:,3),'bo')
hold on
plot3(fingers(:,1),fingers(:,2),fingers(:,3),'ro')
plot3(mcp(:,1),mcp(:,2),mcp(:,3),'co')
plot3(pa_ccw(1), pa_ccw(2), pa_ccw(3),'mx')
plot3(pb_ccw(1), pb_ccw(2), pb_ccw(3),'bx')
plot3(pb_cw(1), pb_cw(2), pb_cw(3),'b+')
plot3(pc_cw(1), pc_cw(2), pc_cw(3),'cx')
plot3(pc_ccw(1), pc_ccw(2), pc_ccw(3),'c+')
plot3(pd_cw(1), pd_cw(2), pd_cw(3),'gx')
plot3(pd_ccw(1), pd_ccw(2), pd_ccw(3),'g+')
plot3(pe(1), pe(2), pe(3),'rx')
axis([-2 2 -2 2])
xlabel('x')
\begin{verbatim}
260  ylabel('y')
261  zlabel('z')
\end{verbatim}