

Chapter 2

DESIGN
TECHNIQUES FOR
APPROXIMATION
ALGORITHMS

Problem 2.4: Minimum Bin Packing

INSTANCE: Finite multiset I of rational numbers $\{a_1, a_2, \dots, a_n\}$ with $a_i \in (0, 1]$ for $i = 1, \dots, n$ (in a multiset the same number may appear more than once).

SOLUTION: A partition $\{B_1, B_2, \dots, B_k\}$ of I such that $\sum_{a_i \in B_j} a_i \leq 1$ for $j = 1, \dots, k$.

MEASURE: The cardinality of the partition, i.e., k .

A simple sequential algorithm for MINIMUM BIN PACKING, called *Next Fit*, processes the items one at a time in the same order as they are given in input. The first item a_1 is placed into bin B_1 . Let B_j be the last used bin, when the algorithm considers item a_i : *Next Fit* assigns a_i to B_j if it has enough room, otherwise a_i is assigned to a new bin B_{j+1} .

Theorem 2.8 ► Given an instance x of MINIMUM BIN PACKING, *Next Fit* returns a solution with value $m_{NF}(x)$ such that $m_{NF}(x)/m^*(x) \leq 2$.

PROOF The proof estimates the value of the optimal and approximate solutions as functions of the sum of the item sizes, denoted by A (i.e., $A = \sum_{i=1}^n a_i$).

Observe that the number of bins used by *Next Fit* is less than $2\lceil A \rceil$: this is due to the fact that, for each pair of consecutive bins, the sum of the sizes of the items included in these two bins is greater than 1. On the other hand, since the number of bins used in each feasible solution is at least the total size of the items, we have that $m^*(x) \geq \lceil A \rceil$. It follows that

QED $m_{NF}(x) \leq 2m^*(x)$.

Example 2.4 ► The bound stated in Theorem 2.8 is asymptotically tight. In fact, for each integer n , there exists an instance of $4n$ items and an ordering of these items such that $m^*(x) = n + 1$ and $m_{NF}(x) = 2n$. The instance and the order of the items are as follows: $I = \{1/2, 1/2n, 1/2, 1/2n, \dots, 1/2, 1/2n\}$ (each pair is repeated $2n$ times). Figure 2.3 shows both the optimal and the approximate solution found by *Next Fit*.

An obvious weakness of *Next Fit* is that it tries to assign an item only to the last used bin. This suggests a new algorithm, called *First Fit*, that processes items in the input order according to the following rule: item a_i is assigned to the first used bin that has enough available space to include it; if no bin can contain a_i , a new bin is opened.