Numerical Complexity Analysis of Weak Approximation of Stochastic Differential Equations

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Abstract

The thesis consists of four papers on numerical complexity analysis of weak approximation of ordinary and partial stochastic differential equations, including illustrative numerical examples. Here by numerical complexity we mean the computational work needed by a numerical method to solve a problem with a given accuracy. This notion offers a way to understand the efficiency of different numerical methods.

The first paper develops new expansions of the computational error for Itô stochastic differential equations using Malliavin calculus. These expansions have a computable leading order term in a posteriori form, and are based on stochastic flows and discrete dual backward problems. Beside this, these expansions lead to efficient and accurate computation of error estimates and give the basis for adaptive algorithms with either deterministic or stochastic time steps. The second paper proves convergence rates of adaptive algorithms for Itô stochastic differential equations. Two algorithms based either on stochastic or deterministic time steps are studied. The analysis of their numerical complexity combines the error expansions from the first paper and an extension of the convergence results for adaptive algorithms approximating deterministic ordinary differential equations. Both adaptive algorithms are proven to stop with an optimal number of time steps up to a problem independent factor defined in the algorithm. The third paper extends the techniques to the framework of Itô stochastic differential equations in infinite dimensional spaces, arising in the Heath Jarrow Morton term structure model for financial applications in bond markets. Error expansions are derived to identify different error contributions arising from time and maturity discretization, as well as the classical statistical error due to finite sampling.

The last paper studies the approximation of linear elliptic stochastic partial differential equations, describing and analyzing two numerical methods. The first method generates iid Monte Carlo approximations of the solution by sampling the coefficients of the equation and using a standard Galerkin finite elements variational formulation. The second method is based on a finite dimensional Karhunen-Loève approximation of the stochastic coefficients, turning the original stochastic problem into a high dimensional deterministic parametric elliptic problem. Then, a deterministic Galerkin finite element method, of either $h$ or $p$ version, approximates the stochastic partial differential equation. The paper concludes by comparing the numerical complexity of the Monte Carlo method with the parametric finite element method.

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