Numerical Methods for some Fully Nonlinear Elliptic Equations

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The main goal of this presentation is to discuss the numerical solution of fully nonlinear elliptic equations (in the sense of Caffarelli-Cabré), a prototypical one being the celebrated Monge-Ampère equation,

\[
\det D^2 \psi = f \quad \text{in } \Omega,
\]

completed by boundary conditions (such as Dirichlet’s). In (1), \(D^2 \psi\) denotes the Hessian of the unknown function \(\psi\). In order to solve (1), and related equations, we advocate a least squares method, in an appropriate Hilbert space; combined with mixed finite element approximations and preconditioned conjugate gradient algorithms, this approach reduces the solution of the above problems to the solution of a sequence of Poisson problems and of small dimension nonlinear problems (one per grid point, typically). Incidentally, our approach provides an alternative to viscosity solutions. The results of numerical experiments will be presented; they concern, among other problems, the solution of (1) and of the following Pucci’s equation

\[
\alpha \lambda^+ + \lambda^- = 0 \quad \text{in } \Omega,
\]

where, in (2), \(\alpha \in (1, +\infty)\) and \(\lambda^+\) (resp., \(\lambda^-\)) is the largest (resp., the smallest) eigenvalue of \(D^2 \psi\).