KTH Computer Science
and Communication

Numerical Methods for Partial Differential Equations, 7.5 ECTS

Homework 1

1. Prove stability of the Lax–Wendroff scheme

$$u_j^{n+1} = u_j^n + kD_0u_j^n + \frac{k^2}{2}D_+D_-u_j^n,$$

for a periodic problem, under some CFL condition.

2. Show that the ADI scheme

$$\left(1 - \frac{k}{2}D_{+x}D_{-x}\right)\left(1 - \frac{k}{2}D_{+y}D_{-y}\right)u_j^{n+1} =$$

$$\left(1 + \frac{k}{2}D_{+x}D_{-x}\right)\left(1 + \frac{k}{2}D_{+y}D_{-y}\right)u_j^n,$$

for the 2D heat equation with periodic boundary conditions is unconditionally stable.

3. Suppose Q is a semibounded operator. Show that the backward Euler approximation

$$u_j^{n+1} = u_j^n + kQu_j^{n+1},$$

is unconditionally stable.

4. Show that the central difference approximation

$$a(x_j)D_0,$$

of the variable coefficient operator $a(x)\partial_x$, is semibounded for the periodic problem if $a(x)$ is Lipschitz continuous.

These exercises are due Nov 10.