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Adaptive Algorithms for Deterministic and Stochastic Differential Equations

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Abstract

Adaptive methods for numerical solutions of differential equations are useful to combine the two goals of good accuracy and efficiency. This thesis contributes to the theoretical understanding of optimal convergence rates of adaptive algorithms. In particular, this work studies stopping, accuracy and efficiency behavior of adaptive algorithms for solving ordinary differential equations (ODEs), partial differential equations (PDEs) and Itô stochastic differential equations (SDEs).

The main ingredient of the adaptive algorithms is an error expansion of the form "Global error = $\sum$ local error $\cdot$ weight + higher order error", with computable leading order terms. The algorithm uses additional computational work to obtain the error estimate because of the weights. However, the approximation of the weights is required to inform where to refine the mesh to achieve optimal efficiency. Based on the a posteriori error expansions with "error indicator := |local error $\cdot$ weight|", the adaptive algorithm subdivides the time steps or elements if the error indicators are greater than TOL/N, and stops if all N time steps or elements have sufficiently small error indicators. Similar algorithms are derived with either stochastic or deterministic time steps for weak approximations of SDEs including stopped diffusions using Monte Carlo Euler method.

There are two main results on efficiency and accuracy of the adaptive algorithms. For accuracy, the approximation errors are asymptotically bounded by the specified error tolerance times a problem independent factor as the tolerance parameter tends to zero. For efficiency, the algorithms decrease the maximal error indicator with a factor, less than 1, or stop with asymptotically optimal number of final time steps or elements. Note that the optimal here refers to error densities of one sign, i.e. possible cancellation of the errors is not taken into account. For a p-th order accurate method, the $L^{\frac{1}{p+1}}$ quasi-norm of the error density is a good measure of the convergence rate for the adaptive approximation, while the number of uniform steps is measured by the larger L^1-norm of the error density.

This analysis of convergence rates of the adaptive algorithms is based on the convergence of an error density, which measures the approximation error for each time step or element. In particular, the error density is proven to converge pointwise on structured adaptive meshes allowing hanging nodes for tensor finite elements and to converge *almost surely* for SDEs as the error tolerance tends to zero.

Finally, numerical experiments illustrate the behavior of the adaptive methods and show that adaptive methods can be more efficient than methods with uniform step sizes.

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Key words and phrases. adaptive methods, mesh refinement algorithm, a posteriori error estimate, dual solution, computational complexity, stopped diffusion, finite element method, Monte Carlo method

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