Experiment 2:
Gaussian Beams and Interferometers

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February 8, 1996

1 Introduction

The experiment was done on Monday January 29, 1996. we were the first group doing this experiment is this years class. As usual Yenchieh Huang was of great help to get everything running. One major breakdown that slowed us down for many hours was that the plotters failed to work with the HP digital scopes, so a new scope had to be found. The student TA, Dong Zheng helped us to setup the etalon, eventhough we failed to obtain adequate date and had to redo it Thursday morning. Everything seemed to be working much better then.

The lab gave a good view of how Gaussian beams propagates and how it can be measured. I treasure the learning of the different methods of waist measurement, especially the convenient use of BeamView Analyzer.

2 Lens focal length

2.1 Measure by focusing a distant object

A rough method to determine the focal length of a lens is to focus a point source located at a distance much greater than the focal length of the lens.

We measured the 100 mm focal length lens with a florcent tube ceiling light at a distance of approximately 3 m. The error we expect in such a measurement is completely determined by the measuring of the distance from the lens to the focus spot. We assume that we were able to get an accuracy of 10%. The displacement due to finite distance to the lamp source is as low as $f/d_0 = 3\%$ and doesn't affect the values much. Because the tubes are long, we got a line instead of spot.

The measurement was done with two types of lenses, carried out with a normal old fashioned ruler. We obtained a focus length of $90 \pm 10\%$ mm for a single element lens and $100 \pm 10\%$ mm for the double element lens.

2.2 Measure by knife edge test

The knife edge test is a more exact method to measure the focal length. In this measurement we uses a laser with specifications, $w_0 = 408$ μm and $\lambda = 632.8$ nm. This beam was expanded close to the laser with a 20x beam expander. The outgoing waist is then 8160 μm. This give a Rayleigh length of

$$z_R = kw_0^2/2 = 2\pi w_0^2/(2\lambda) = \pi w_0^2/\lambda = 331\ m, \tag{1}$$

which is a little bit more distance than we can get inside Ginzton Labs without reflecting the beam. Because $z_R \gg f$ we can use geometrical optics formulas, so the spot will focus in at the focal length.

After the expander the beam goes through the lens and focuses at the focal length distance from the lens. The beam is then projected at a white screen at a distance of around 2 focal lengths from the lens. A razor-blade fixed to a position adjustable block was carefully introduced at one side of the beam-path between the lens and the screen.

If the razor located before the focus, it cuts the beams traveling to the screen side opposite to the razor side. The laser profile on the opposite side will then disappear.

If the razor instead is located after the focal point, the light on going to the razor side of the screen is blocked.

At the very spot the beam profile disappeared all at once. This distance was measured precisely to 100mm for both lenses. [1]

[1] Maybe we were lucky this time

3 Gaussian Beam Spot Size and Divergence

3.1 Theoretical analysis

The expansion of the beam can be derived from classical Maxwell equations, in the following text there is some equations that isn't derived, the derivations can be found in the lecture notes[1] and in Guenters book[2].

3.1.1 Waist expansion

The spatial electrical field of a TEM_{00} is proportional to $e^{-(\rho/w(z))^2}$, therefore the intensity is proportional to $e^{-2(\rho/w(z))^2}$. At the waist, $\rho = w(z)$, we have a intensity $1/e^2$ of peak value. The waist will expand as[1],

$$w^2(z) = w_0^2 \left(1 + \left(\frac{z}{z_R}\right)^2\right), \tag{2}$$

where z_R is the Rayleigh length given in equation (1) and w_0 is the minimum waist.

3.1.2 Waist measure vs distance with chopper

Theoretical beam waist from the He-Ne laser is approximately 408 μm, we use a chopper wheel with one radial slit of 10 μm at radius, $r = 37\ mm$, to scan over the laser beam and measure the light output spatially. Because the slit is only a fraction of $w_0 = 425\ \mu m$, it's not necessary to integrate the beam profile in x (i.e ϕ) direction of the slit. But in the radial direction the throughput is integrated over the radial length of the slit. Because the length of the slit is large in comparison to the beam waist, the integration can be carried out to infinity. So the intensity of the throughput is,

$$I(x) \propto \int_{-\infty}^{+\infty} e^{-2(x^2+y^2)/w(z)^2)} dy \propto e^{-2(x/w(z))^2}.$$

The slit cuts the laser beam at a speed of $v = r\omega = 2\pi\nu r$. The spatial coordinate, x, will depend on t as $x = vt = 2\pi\nu rt$. If we denote the distance between the $1/e^2$ values as $\Delta x = 2w(z)$. Then $\Delta x = 2\pi\nu r\Delta t$. Isolating w(z), we get $w(z) = \pi\nu r\Delta t$, where Δt of course is the time between $1/e^2$ values.

Table 1: Measured Data for Chopper and BeamView

Distance [cm]	*Chopper [μm]*
20.32	446
40.64	479
60.96	507
80.01	581
99.06	700
119.30	825
139.70	900

3.2 Measure of waist vs distance without lens

The throughput intensity is measured with New focus detector model 2001 Data was obtained at 7 different distances from the laser ranging from 20.32 cm to 139.70 cm. The odd numbers come from the inch distances on the laboratory table. Some of the data from the digital oscilloscope was plotted(Plot 3.I-V). The processed data is presented in table 1.

3.2.1 Curve-fitting

According to equation (2) the waist can be written as a second order polynomial. We fit the squared waist to $az^2 + bz + c$ and from these estimates we can after some algebra obtain,

$$w_0 = \sqrt{c - b^2/4a} = 442\ \mu m$$

and

$$z_0 = -b/2a = 18\ cm$$

Plotting the result in figure 1, we see that we have a reasonable fit. The waist seem to be within reasonable values, but it really don't make sense that the waist is located outside the laser. One reason could be that the beam is semi-Gaussian and broadens faster. Therefore the curve-fitting would intersect the minimum waist farther out(large positive z_0).

The divergence of the beam can be calculated with to,

$$\theta = \lambda/\pi w_0 = 0.456\ mrad$$

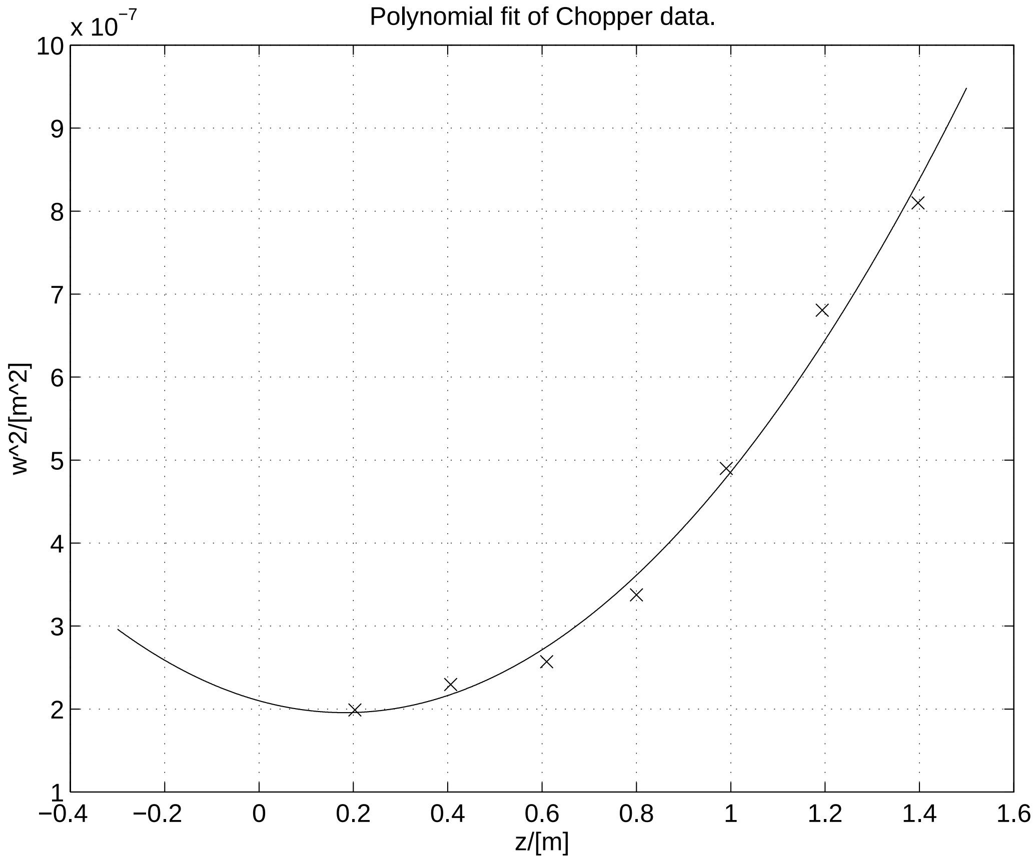

Figure 1: Measured data from chopper, The measured points are fitted to a second order polynomial

If we instead uses the last three data points we get a divergence of 0.430 $mrad$.

3.3 Beam waist with focusing lens

This part of the lab, we wanted to measure the waist at focus of a lens. We first expand the beam from the laser with a 20x beam expander. This results in a 20 times wider waist, $w_1 = 20w$. We use w from the previous experiment, the setup was done at a distance of 20 cm. The broad beam is then focused in a lens element and the focus waist in measured with a chopper as in the previous experiment. For laboratory setup, we used a position movable block with the chopper mounted. We placed the photo detector very close to the chopper, to be able to detect the weak signal. Because of limited space, we used the much smaller FDS100 instead of Mod.

2001. We used the 50Ω input of the scope as resistance. Because of the position block, we could place the chopper in focus with good accuracy. Focus was attained when a small change in either direction give an increase in waist.

The theoretical diffraction limit of the waist at focus for a 10 cm lens with a incident beam-width of 20×446 μm is [1],

$$w_{focus} = \frac{f\lambda}{\pi w} = \frac{1 \times 10^{-1} * 632.8 \times 10^{-9}}{\pi 20 \times 446 \times 10^{-6}} = 2.3\ \mu m$$

But in practice we cannot obtain this value. With the experimental setup for the double-element lens, with the thick side to the laser, we obtained a waist of 8.5 μm. Putting thr lens in the other direction we got 6.6 μm (Plot 3.VI-VII). The TDL(Times Diffraction Limited) is 3.7 and 2.9 respectively.

Second we used a singlet lens. We obtained waists of 8.9 μm with the rounded face against the laser. Opposite we get 9.1 μm (Plot 3.VIII-IX). The TDL is here 3.9 and 4.0 respectively.

The high values for TDL comes from spherical aberration that give a more inferior resolution. The singlet are not compensated for spherical aberrations as good as for the doublet, which implies a greater TDL.

3.4 Using an aperture to look at changes due to spherical aberration

This part of the lab was added because we wanted to see how the TDL decrease as the input beam was made thinner wither a aperture. Most of the spherical aberration is in the outer part of the lens. Unfortunately I didn't obtain any data, we played with it a bit and saw some hint of this when we used the singlet. Maybe this could be included in next years lab.

4 BeamView Analyzer

4.1 Measure of laser spot size vs distance

Using the computer aided tool, BeamView, we were able to get the beam profile presented on a computer screen. We used a variable ND-filter to attune the laser power to get a unsaturated picture from the CCD-camera. The beam profile was

Table 2: Measured Data for BeamView for comparison to chopper data

Distance [cm]	*Chopper [μm]*	*BeamView [μm]*
20.32	446	457
40.64	479	454
60.96	507	459
80.01	581	469
99.06	700	484
119.30	825	503
139.70	900	526

measured at the same distance as we did with the chopper. The CCD-camera was very easy to use and the measurement was done without any problem. The different beam profiles we got can be seen in Plot 4.I. The actual data in presented in table 2.

As we did for the chopper, we can use a second-order polynomial and fit the data points, the resulting plot can be seen in figure 2. The waist and minimum waist location is calculated to be,

$$w_0 = \sqrt{c - b^2/4a} = 455\ \mu m$$

and

$$z_0 = -b/2a = 34.5\ cm.$$

Again we see the problem with z_0 being outside the laser. Also we want to calculate the beam divergence, which is

$$\theta = \lambda/\pi w_0 = 0.442 mrad.$$

If we instead interpolate the last three data points we get a divergence of 0.108 $mrad$.

4.2 Compare to results measured by the chopper

Comparing the data obtained in table 3 we see some big differences between the two measuring methods. The measurement for the chopper is closer to theory than the BeamViewer. Maybe the the BeamView was saturated when we measure at shorter

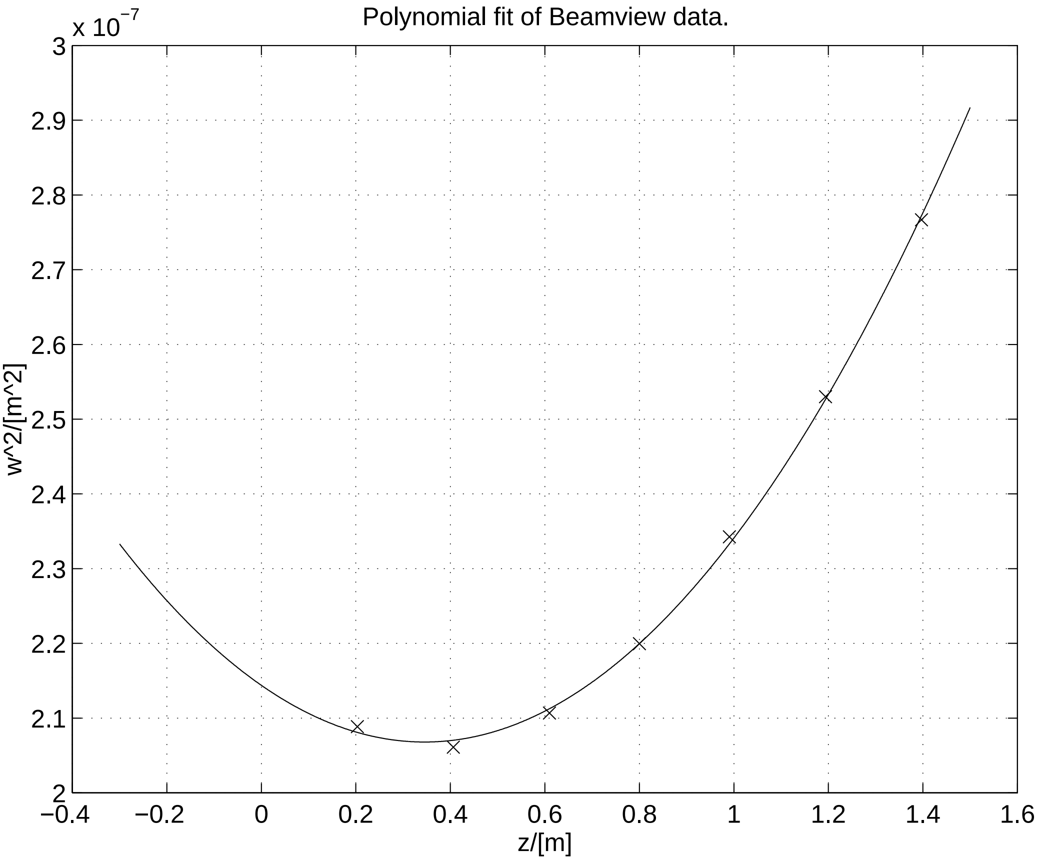

Figure 2: Plot of the squared waist vs distance for BeamView Analyzer

Table 3: Summary of data from Chopper and BeamView.

Attribute	*Chopper*	*BeamView*	*Theoretical*
$w_0[\mu m]$	442	455	408
$z_R[cm]$	18	34.5	~ 0
calc. $\theta[mrad]$	0.456	0.442	0.493
inter. $\theta[mrad]$	0.430	0.108	N.A.

distances. Maybe the ND-filter changed the beam profile. Anyhow, it gives a rough estimation of w_0. The huge z_R can come from the non-Gaussian shape in the near field.

5 Etalon

The etalon can be used to scan a spectrum from a laser etc. In this part we want to analyze the finesse of the etalon. The finesse is defined as the distance between the peaks and divided by the FWHM intensity of the peaks

5.1 Theoretical analysis

An etalon is a thick glass piece that function as a cavity. The cavity give most transmittance when the phase of all cavity reflections correspond. This happens when the the optical path difference between these reflections is a multiple of λ. If they instead is a odd multiple of $\lambda/2$ there is a phase cancellation and the etalon works more like a reflector. If the etalon is tilted the optical path will lengthen. The full derivation of transmittance gives[1]

$$n\sin(\theta_2) = sin(\theta_0) \tag{3}$$

$$\Phi = \frac{4\pi}{\lambda} nd\cos(\theta_2) \tag{4}$$

$$F = 4R/(1-R)^2 \tag{5}$$

$$T_{tot} \propto \frac{1}{1+F\sin^2(\Phi/2)} \tag{6}$$

where θ_0 is the incident angle, θ_2 is the angle inside the etalon with index of refraction of n. R is the power reflectivity of the etalon. T_{tot} is the total transmittance of the etalon. As we see the FWHM changes with θ_0, we want to be able to extract F out of experimental results, so we have to find the relationship between FWHM, $\Delta\theta_0$ and θ_0. Half intensity occur when the denumerator of equation 6 is equal to 2, in other words when

$$F\sin^2(\Delta\Phi/4) = 1$$

where $\Delta\Phi/2$ is the change from the peak to the FWHM. equation 3 gives with Taylor expansion for small θ_2,

$$\Phi \approx \frac{4\pi}{\lambda}nd(1-\theta_2^2/2) \tag{7}$$

$$\Delta\Phi = \frac{4\pi d\theta_2\Delta\theta_2}{\lambda} \tag{8}$$

$$F = \sin^{-2}\left(\frac{\pi d\theta_2\Delta\theta_2}{\lambda}\right) \tag{9}$$

$$\mathcal{F} = \pi\sqrt{F}/2 = \frac{\pi}{2}\sin^{-1}\left(\frac{\pi d\theta_2\Delta\theta_2}{\lambda}\right) \tag{10}$$

5.2 Simulation

A simulation of equation (6) was done in MATLAB. Using the values $d = 2mm$, $F = 16$. The result can be seen in figure 3. This data can be used to compare with the experimental ones.

5.3 Measurement

The etalon was measured using a galvo that give an angle change proportional to the voltage applied. We drove the galvo with a triangle wave with peak voltage of 8 volts. Because there is some inertia in the galvo and etalon we couldn't use higher frequencies than 4 Hz. The outgoing beam was detected using the Mod. 2001 and the internal 50Ω resistance inside the digital oscilloscope. The total angular change of the etalon was measured by looking at the reflexions on the wall. The etalon was 109 in. from the wall and gave a pattern than was 6.375 in. long. By looking at the plot (Plot 5.I) of the transmittance on the scope, I draw conclusion that the lowest point of triangle waveform represents $\theta_0 = 0$. The maximum θ_0 can then be calculated to,

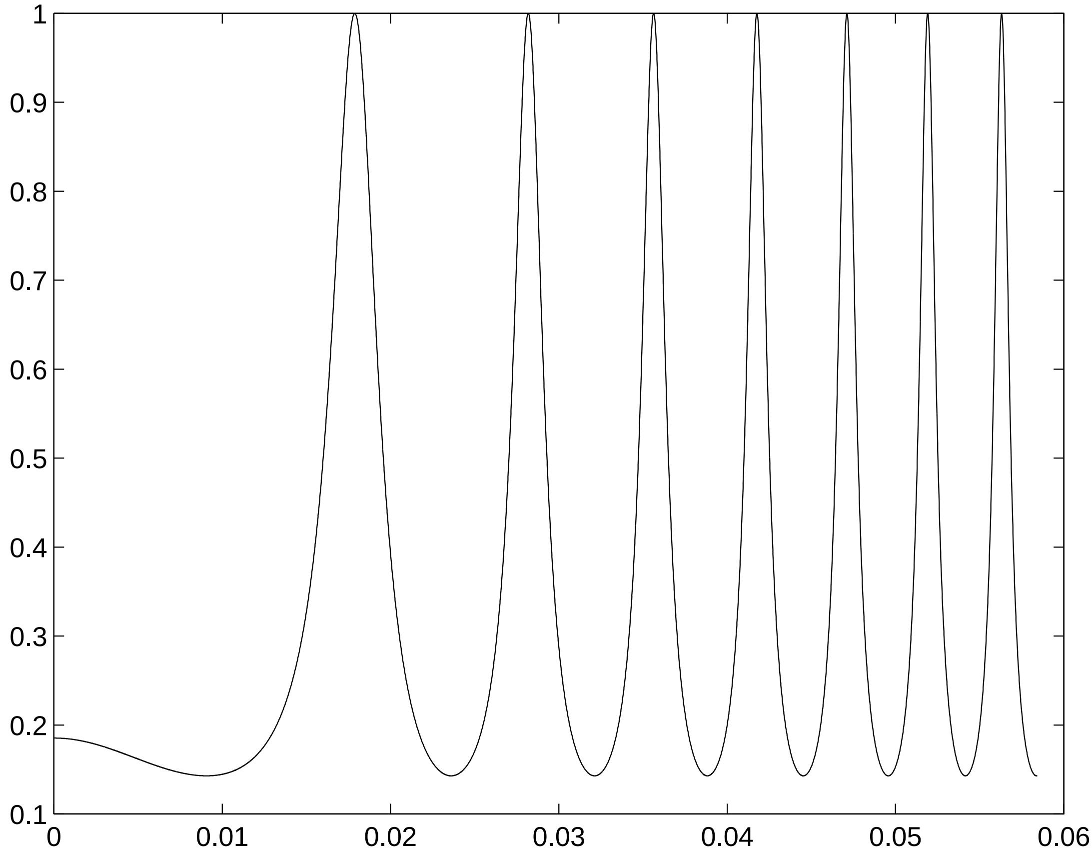

Figure 3: Simulation of transmittance of a 2mm etalon with F=16.

$$\theta_{0max} = \tan^{-1}(6.375/109) = 5.84 \times 10^{-2}\ rad$$

So the voltage sensitivity of the galvo is $5.84 \times 10^{-2}/8\ rad/V = 7.30 \times 10^{-3}\ rad/V$. The measurement of the transmittance was plotted (Plot 5.I). It correspond very well with the simulated result in figure 3. So the voltage sensitivity of the galvo is $5.84 \times 10^{-2}/8\ rad/V = 7.30 \times 10^{-3}\ rad/V$. The etalon type used have a thickness of 2mm. We can calculate the finesse using the first peak at $\theta_0 = 1.34 \times 10^{-2}$ with a $\Delta\theta_0 = 3.3 \times 10^{-3}$. Assuming that the etalon has a index of refraction of 1.5, it results in $\theta_2 = 8.93 \times 10^{-3}$ and $\Delta\theta_2 = 1.86 \times 10^{-3}$. Using the equation (10) we get a finesse of 6.41. This seem to be very low, but by doing to approximative measurement in the plot gave ~ 5. This etalon isn't very good or all my calculations are wrong.

6 Scanning Confocal Interferometer

6.1 Intentions

One very easy way to get the axial modes of a laser is by using a confocal interferometer. In this experiment we use one with a span of 2 GHz for the He-Ne spectra.

By changing the length of the confocal cavity by applying a sawtooth voltage to a mirror displacer in the interferometer we can scan the over the frequency of input beam.

6.2 Theory

The axial modes spacing in a cavity is[3],

$$\nu_a x = \frac{c}{2L} \tag{11}$$

because the scan was 2 GHz the internal effective cavity length can be calculated to 75 mm.

6.3 Measure the He-Ne laser optical spectrum

We directed the laser beam into the interferometer and spent some good time to align it. The sweep sawtooth and the signal was connected to the scope. Plot 6.I shows three laser peaks that is repeated once. So the difference is frequency for the periodicity is 2GHz. The left axial mode is suppressed because of some asymmetry reason. Anyhow they are all axial modes, but we see clearly that the one in the middle has the highest gain.

Because we know the distance between the two repeated groups is 2 GHz, We can get the distance between the axial modes by using ratios. The distance between the axial modes is then 432 MHz. The length can be calculated with equation (11) to be 347 mm. This can the compared to the specified cavity length of 345 mm. Fairly close I would say.

If we assume that the line-width of the laser is small in comparison to the broadening by the confocal interferometer. The width of the laser-lines are about 24 MHz. So the finesse would be 2GHz/24MHz=82.

References

[1] Prof. Marty Fejer *Lecture notes AP304 Stanford Univ. 1996*

[2] Robert Guenter *Modern Optics*, Wiley 1990

[3] Anthony E. Siegman *Lasers* University Science Books 1986

[4] Prof. Marty Fejer *Laboratory Instructions Exp. 2* Stanford 1996