FAULT-TOLERANT FITTING AND ONLINE DIAGNOSIS OF FAULTS IN SISO PROCESS

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Abstract：In this paper, a series of recursive fault-tolerant (FT) fitting algorithms are built for the trajectories of a SISO process when some pulse-type faults arise from output components of the process. Based on the recursive FT fitting, a series of practical program are given to online detect pulse-type faults in process and to identify magnitudes of these faults. Simulation results show that these new methods are efficient.

Keywords：Fault-Tolerance**,** Fault Detection**,** Fault Identification, Non-stationary Signals, Sensors

1. INTRODUCTION

In many fields such as process safety and dynamic system surveillance, it is a valuable and widely applicable task to detect whether there exist any faults occurred at a running process or not and to conclude the magnitudes of these faults. In order to detect and diagnose faults of a process or a system, there were many kinds of approaches built, such as detecting filters, analysis of influence function, residual analysis, parity equation and parity space, probability ratio or generalized likelihood ratio, innovation analysis of Kalman filter and classical statistical diagnosis, etc. Patton et al (1989) and Isermann (1984) and Hu & Sun (1999a) have summarized the evolution of fault detection and diagnosis (FDD) from different viewpoints.

Summing up recent reference on FDD, we may find out that most of the approaches were based on the least squared (LS) estimators of process parameters, the LS fitting of process trajectories, likelihood ratio function, or equivalent transformation of process model, etc. Generally, these approaches stated above possess excellent properties when the process runs properly. So, these detection algorithms can be efficiently used to detect online the first fault and to detect offline more than one faults arisen at different times, when a dynamic process runs with fault.

But, a lot of theoretical research and practice open out that some classical statistics (such as the LS estimators, the Kalman filter and the likelihood ratio, etc) lack of the fault-tolerance against the bad influence from faults arisen before the monitoring time**.** In other words, these detection algorithms have bad behaviors and may make false decision (see Isermann,1984;Hu,1993;Hu and Sun, 1999b) in online monitoring the process that has historical faults.

In engineering fields, when a dynamic process run for a long time, it is possible that there may exist some faults occurred at different component in different times in the running process. In this case, it is important to discuss how to online monitor the state of the dynamic process with historical faults.

In this paper, we build a series of practical algorithms to detect faults online and to estimate magnitudes of the faults. These algorithms have strong fault-tolerance to faults and can be reliable to monitor type-pulse faults. Some simulations will show the validity of these new algorithms given in this paper.

2. THE RECURSIVE FT FITTING

Assume that the output of a continuous variable dynamic system is a measurable stochastic process $\{y(t), t \in T\}$ and that the expectant trajectory $\{\mu(t), t \in T\}$ of the output is square integrable and smooth piecewise in all of the finite intervals. Basing on the Weierstrass theorems and their generalized results (see Hu and Sun,1998), the function $\mu(t)\,(t \in [t_a, t_b] \subseteq T)$ can be approximated by the linear combination of a series of reasonably selected base $\{x_j(t) \in L^2[t_a, t_b], j = 1,2,3,\cdots\}$:

$$\sum_{j=1}^{s} a_j x_j(t) \to \mu(t) \quad (s \to +\infty) \qquad \textbf{(1)}$$

where the $\{a_j \in R, j = 1,2,3,\cdots,s\}$ are real constants.

Considering the compensating error principle, the formulae (1) can be expressed as following

$$y(t) = \sum_{j=1}^{s} a_j x_j(t) + \varepsilon(t) \qquad \textbf{(2)}$$

where the process error $\{\varepsilon(t), t \in T\}$ is integration of stochastic noise and error from fitting model.

Let us construct a vector $\vec{\alpha} = (a_1, \cdots, a_s)^\tau$. The key problem is how to identify the coefficients' vector $\vec{\alpha}$ when we use the model (2) to fit output of the monitored process.

The classical identification statistics of $\vec{\alpha}$ are the LS estimators and the recursive LS estimators.

2.1 The Recursive LS Fitting

In many engineering fields, dynamic processes run for a long time generally and involve complicated dynamic properties. In order to online analysis and to online predict state of the process, the recursive fitting is very necessary.

Now, let's build a series of recursively gliding LS estimators for the coefficients $\vec{\alpha}$ of the fitting model. Using the following notation

$$\begin{cases} H_{(i\to j)} = \begin{bmatrix} x_1(t_i) & \cdots & x_s(t_i) \\ \vdots & & \vdots \\ x_1(t_j) & \cdots & x_s(t_j) \end{bmatrix} \quad (1 \le i \le j) \\ \vec{h}_i = \begin{bmatrix} x_1(t_i) \\ \vdots \\ x_s(t_i) \end{bmatrix}, \quad Y_{(i\to j)} = \begin{pmatrix} y(t_i) \\ \vdots \\ y(t_j) \end{pmatrix} \end{cases} \qquad (3)$$

it can be proved that, basing on the measurement $D_{(i+1\to i+n)} = \{y(t_{i+1}), \cdots, y(t_{i+n})\}$, the LS estimators of $\vec{\alpha}$ can be expressed as the following

$$\hat{\vec{\alpha}}_{LS(i+1\to i+n)} = J_{(i+1\to i+n)} H^\tau_{(i+1\to i+n)} Y_{(i+1\to i+n)} \qquad (4)$$

where $J_{(i+1\to i+n)} = (H^\tau_{(i+1\to i+n)} H_{(i+1\to i+n)})^{-1}$, $n \ge s$.

With the system's running up progressively, the formula (4) satisfy a recursive algorithm as following

$$\hat{\vec{\alpha}}_{LS(1\to n+1)} = \hat{\vec{\alpha}}_{LS(1\to n)} + p_{(n+1|n)} \tilde{f}_{LS(n+1|n)} \qquad (5)$$

where $\tilde{f}_{LS(n+1|n)} = \dfrac{y(t_{n+1}) - \vec{h}^\tau_{n+1} \hat{\vec{\alpha}}_{LS(1\to n)}}{(1 + \vec{h}^\tau_{n+1} (H^\tau_{(1\to n)} H_{(1\to n)})^{-1} \vec{h}_{n+1})^{1/2}}$ and $p_{(n+1|n)} = \dfrac{(H^\tau_{(1\to n)} H_{(1\to n)})^{-1} \vec{h}_{n+1}}{\sqrt{1 + \vec{h}^\tau_{n+1} (H^\tau_{(1\to n)} H_{(1\to n)})^{-1} \vec{h}_{n+1}}}$.

From the formula (4), if the process runs properly we can conclude that the recursive LS fitting of the trajectories at time t is equal to

$$\hat{y}_{LS(1\to 1+n)}(t) = \sum_{j=1}^{s} \hat{a}_{j,LS(1\to 1+n)} x_j(t) \qquad (6)$$

Correspondingly, the predictor one-step ahead of the process output $y(t_{n+1})$ is $\hat{y}_{LS(1\to n)}(t_{n+1})$. Let's note the error of the predictor one-step ahead is as following

$$\hat{\varepsilon}_{LS(n+1|1\to n)} = y(t_{n+1}) - \hat{y}_{LS(1\to n)}(t_{n+1}) \qquad (7)$$

Theorem 1 If the mean and variance of the dynamic noise $\{\varepsilon(t_i) : i = 1,2,\cdots\}$ are equal to zero and σ^2 respectively, then variance of the error $\hat{\varepsilon}_{LS(n+1|1\to 1+n)}$ is equal to

$$Var(\hat{\varepsilon}_{LS(n+1|1\to 1+n)}) = (1 + \vec{h}^\tau_{n+1} J_{(1\to 1+n)} \vec{h}_{n+1}) \sigma^2 \qquad (8)$$

Proof: Combining the LS estimators $\hat{\vec{\alpha}}_{LS(1\to 1+n)}$ with (6) into formulae (7) and do some reduction, we can obtain the formula (8). ▌

2.2 Fault-tolerance Improvement of the LS Fitting

The widely used recursive estimators (5) are recursive minimizing-variance unbiased (UMVU) estimators of the fitting coefficients when the process runs properly and exports output without faults. So, the recursive LS fitting possess excellent statistical properties.

On the other hand, the recursive LS fitting has the same weakness as the ordinary LS fitting: the practical fitting results are unsatisfactory and can break down in the case there exists abrupt faults occurred at the running process. In fact, If there exist a fault occurred at time t_0, whose magnitude is $\lambda(t_0)$, namely

$$\tilde{y}(t) = \begin{cases} y(t), & t \neq t_0 \\ y(t) + \lambda(t), & t = t_0 \end{cases} \tag{9}$$

then the error of the recursive LS predictor one-step ahead can be expressed as following

$$\hat{\hat{\varepsilon}}_{LS(n+1|1\to 1+n)} = \hat{\varepsilon}_{LS(n+1|1\to 1+n)} + \begin{cases} 0, & t_0 > t_{n+1} \\ \omega(t_{n+1}, t_s), & t_1 \le t_0 < t_{n+1} \\ \lambda(t_{i_0}), & t_0 = t_{n+1} \end{cases} \tag{10}$$

where $\omega(t_{n+1}, t_0) = -\vec{h}^{\tau}_{n+1}(H^{\tau}_{(1\to 1+n)} H_{(1\to 1+n)})^{-1}\vec{h}_0$.

The expression (10) shows that pulse-type faults occurred at a running process may result in evident magnifying the predictor error, the numerical value of which are not equal to any constants at different gliding interval. If we view the $\hat{\hat{\alpha}}_{LS(1\to 1+n)}$ in formulae (5) as a modification of $\hat{\hat{\alpha}}_{LS(1\to n)}$ by the LS filtering residual (7) and the predicting error $\hat{\varepsilon}_{LS(n+1|1\to n)}$, then we may find out that a pulse-type fault occurred at time t_{n+1} not only unconventionally largen the predictor error $\hat{\varepsilon}_{LS(n+1|1\to n)}$, but also evidently change the estimators $\hat{\hat{\alpha}}_{LS(1\to 1+n)}$ and even break down the algorithm (5).

In order to overcome the bad influence from exceptional change brought by a pulse-type fault on the gliding recursive estimators of model coefficients, we set a re-descending (see Hu and Sun,1999c) ϕ-function as following

$$\phi_{rd}(x) = \begin{cases} x & |x| < c_1 \\ c_1 \, sign\,(x) & c_1 \le |x| < c_2 \\ \dfrac{c_2 - |x|}{c_2 - c_1} c_1 & c_2 \le |x| < c_3 \\ 0 & |x| \ge c_3 \end{cases} \tag{11}$$

where the (c_1, c_2, c_3) are constants, and use this kind of ϕ-function to cut down the bad influence of informal predicting error on the recursive identification algorithms. By ϕ-function (11), we construct an improvement algorithms for formulae (5) as following：

$$\hat{\alpha}_{\phi(1\to n+1)} = \hat{\alpha}_{\phi(1\to n)} + p_{(n+1|n)}\phi(\tilde{f}_{\phi(n+1|n)}) \tag{12}$$

where $\tilde{f}_{\phi(n+1|n)} = \dfrac{y(t_{n+1}) - \vec{h}^{\tau}_{n+1}\hat{\hat{\alpha}}_{\phi(1\to n)}}{(1 + \vec{h}^{\tau}_{n+1}(H^{\tau}_{(1\to n)}H_{(1\to n)})^{-1}\vec{h}_{n+1})^{1/2}}$.

Theorem 2 If noise series $\{\varepsilon(t), t \in T\}$ of model (2) are stationary and their distributions are symmetrical, whose mean and variance are equal to zero and σ^2 respectively, the estimators $\{\hat{\hat{\alpha}}_{\phi(1\to n)}, i = 1,2,3,\cdots\}$ are unbiased in the case there are not any faults occurred before the time t_n and initial value $\hat{\hat{\alpha}}_{\phi(1\to n_0)}$ of the algorithm (12) is selected as $\hat{\hat{\alpha}}_{LS(1\to n_0)}$.

Proof：Distinctly, $\hat{\hat{\alpha}}_{\phi(1\to n_0)}$ is an unbiased estimator of the vector $\vec{\alpha}$. Using the property that the integral of an odd function on symmetrical interval is equal to zero, we can also prove that $\hat{\hat{\alpha}}_{\phi(1\to n_0+1)}$ being unbiased. Similarly, it can be proven that the estimators $\{\hat{\hat{\alpha}}_{\phi(1\to n)}, i = 1,2,3,\cdots\}$ are unbiased by mathematical induction. ▌

Now, let's expatiate on the rationality of the new algorithm (12). Using a term "innovation" (see Mehra and Peschon,1971), we regard the one-step predicting error $\hat{\varepsilon}_{LS(1+n|1\to n)}$ as innovation brought by the measuring data at time t_{1+n}. It is obviously reasonable for us to substitute $\phi(x) = x$ implied in (5) for the re-descending ϕ-function given in (11), because of the following reasons：

- when an innovation of sampling data falls into the anticipative bound, we think that this new sampling data is reasonable and should make full use of this accept use all of this innovation；
- when the innovation of sampling data goes beyond a selected bound but do not overtop a lot, we must use the reasonable influence and confine the bad influence from this sampling data；
- we must escalate the restriction on making use of information from the distrustful sampling data；
- when the innovation distinctly depart from the normal value and is exceptionally large, we may quite eliminate the bad influence from them.

Basing on the explanations stated above, we may find out that the modified recursive algorithms (12) can more reasonably absorb the innovation from sampling data $y(t_{1+n})$ to update the identification of parameters than the estimators (5), when there exist large discrepancy between a sampling data $y(t_{1+n})$ and the predictor $\hat{y}(t_{1+n}) = \vec{h}^{\tau}_{1+n}\vec{\alpha}_{\phi(1\to n)}$. In other words, the FT algorithm (12) have the ability to improve reliability of estimators of parameters by reducing optimality of algorithms. It is compromising between statistical optimality and fault-tolerance.

3. FT DETECTION OF FAULTS IN PROCESS

Generally, if there do not exist any faults before the time t_{1+n}, we have pointed out that the predicting error $\hat{\varepsilon}_{LS(n+1|1\to n)}$ obeys the normal distribution $N(0, d_{(1\to n)}\sigma)$ when the process noise $\{\varepsilon(t), t\in T\}$ is white and stationary Gaussian noise. Basing on this property, we may use the following detecting statistics to diagnose whether there is a fault occurred at time t_{n+1} or not:

$$\Re_{LS}(t_{n+1}) = \frac{y(t_{n+1}) - \vec{h}^{\tau}_{n+1}\hat{\vec{\alpha}}_{LS(1\to n)}}{d_{(1\to n)}} \tag{13}$$

where $d_{(1\to n)} = \sqrt{1 + \vec{h}^{\tau}_{1+n} J_{(1\to n)} \vec{h}_{1+n}}$.

Now, the problem is that the detection statistics can not be used in the case there exist one or more faults occurred before the time t_{n+1} because the detection statistics (13) is based on the LS algorithms and is hypersensitive to outliers (see Hu and Sun,1999c) in sampling data. Namely, the algorithm (13) can not be used in monitoring multiple faults of a dynamic process.

In order to protect a detection statistics against any bad influence from historical faults, it is reasonable to substitute the FT estimators for the LS estimators.

In section 2.2, it has been pointed out that the estimators $\hat{\vec{\alpha}}_{\phi(1\to n)}$ given by formulae (12) have the ability to overcome bad influence from pulse-type faults and to make sure the reliability of estimators. So we replace $\hat{\vec{\alpha}}_{LS(1\to n)}$ in expression (13) with $\hat{\vec{\alpha}}_{\phi(1\to n)}$. This means is a practical approach without failure for online monitoring dynamic process in the case there are multiple faults having occurred before the sampling moment detected.

According to all of the analysis stated above, a series of detecting strategies, which are based on the FT recursive estimators, are built as following：

1). Selecting $n_0 > \min\{s, n-1\}$, let's do some detection offline to diagnose whether there exist any faults before the time t_{n_0} or not with the robust-likelihood ratio detection algorithm given in Hu Shaolin and Sun Guoji (1999c) and correct the outliers by interpolation；

2). The $\hat{\vec{\alpha}}_{LS(n_0-n+1\to n_0)}$ is calculated by (4) and set as the initial value of the recursive algorithm (12)；

3). According to algorithm (12), a series of gliding calculation is done to obtain $\hat{\vec{\alpha}}_{\phi(i+1\to i+n)}$；

4). The detection statistics is done as following

$$\Re_{\phi}(t_{n+1}) = \frac{y(t_{n+1}) - \vec{h}^{\tau}_{n+1}\hat{\vec{\alpha}}_{\phi(1\to n)}}{d_{(1\to n)}} \tag{14}$$

5). A judgement is done：if $|\Re_{\phi}(t_{n+1})| \geq c$, there is a fault occurred at time t_{n+1}; otherwise, the sampling data is normal and the process is running without fault, where the constant c is a bound selected properly (The default value is suggested as 3σ).

6). With the process running continuously, step3)~5) are done again and again.

4. THE FT IDENTIFICATION OF FAULTS

Setting $\vec{h}^{\tau}_{n+1}\hat{\vec{\alpha}}_{\phi(1\to n)}$ as a predicting value of the $y(t_{n+1})$, a statistics is constructed as following

$$\begin{aligned} \hat{\lambda}_{\phi}(t_{n+1}) &= \hat{\varepsilon}_{\phi(n+1|1\to n)} \\ &= \{y(t_{n+1}) - \vec{h}^{\tau}_{n+1}\hat{\vec{\alpha}}_{\phi(1\to n)}\} \end{aligned} \tag{15}$$

to conclude the magnitude of a fault.

Theorem 3 Assuming that the noise process $\{\varepsilon(t), t\in T\}$ is white stationary Gaussian noise, whose mean is zero, the mathematical expectation of statistics $\hat{\lambda}_{\phi}(t_{i+n+1})$ is equal to zero or $\lambda(t_{n+1})$ in the case that the dynamic process runs properly or there exists a fault, whose magnitude is $\lambda(t_{n+1})$, occurred at time t_{n+1} respectively.

Proof： The result of the theorem can be educed by theorem 2 and expression (15) obviously. ▌

5. SIMULATION

The simulation model is selected as the polynomial

$$y(t) = b_0 + b_1 t + b_2 t^2 + \varepsilon(t), \varepsilon(t) \sim \mathrm{N}(0,1) \tag{16}$$

And, setting $b_0 = 100.0, b_1 = 10.0, b_2 = -0.25$, we use the

Monte Carlo method and the model (16) to generate 100 sampling data at time set $\{t_i = ih \mid i = 1, \cdots, 100, h = 1s\}$.

By intentionally shifting the sampling data $(-1)^{i+1}100$ at time set $\{t_i \mid i = 51 \sim 55, 75\}$ respectively, we get a new series of output data with multiple relative faults and an isolated fault as in fig 1.

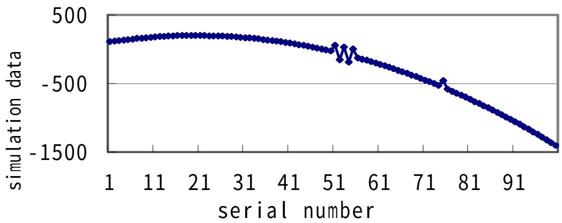

Fig 1. Simulation Data with Two Kinds of Faults

From fig 1, we can find out that the magnitudes of the isolated fault and the relative faults are not prominent.

5.1 The behaviors of FT estimators

Using the sample data in fig 1 and the generating algorithm formula (7) of the recursive LS fitting, we get the plot of the recursive LS predicting residuals which are plotted in fig 2:

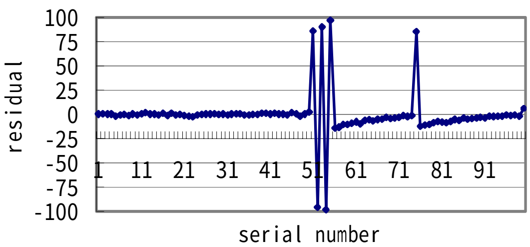

Fig 2. the recursive LS predicting residuals

At the same time, we use the following generation algorithms of the recursive FT predicting residuals:

$$\hat{\varepsilon}_2(t_{n+1}) = y(t_{n+1}) - \sum_{i=1}^{s} \hat{a}_{i,LS(1\to n)} \hbar_i(t_{n+1}) \qquad (17)$$

and plot the FT prediction residuals in fig 3:

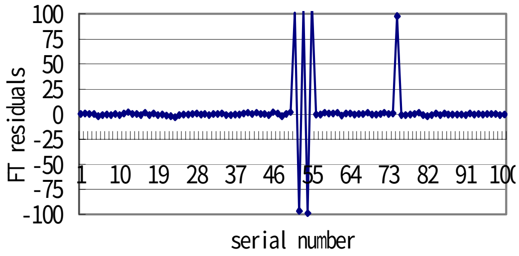

Fig 3. the recursive FT predicting residuals

Comparing the figure 2 with figure 3, we are convinced that the recursive LS estimators of model coefficients are influenced badly by outliers and that the recursive FT algorithm (12) has commendable resistance to outliers.

5.2 Detecting and Identifying Faults

Using the LS prediction residuals plot shown in fig 2 and the FT prediction residuals plot shown in the fig 3 respectively, we obtain two series of different estimators of the fault magnitudes shown in table 1.

From the table 1, we can educe the conclusion that the FT estimators $\hat{\lambda}_\phi(t)$ are markedly closer to the magnitude values of faults that are designed for simulation than the LS estimating values $\hat{\lambda}_{LS}(t)$ $(i_0 = 51 \sim 55, 75)$. So,the detection algorithms given in this paper are reliable to detect faults and to identify fault magnitudes.

Table 1. The LS estimators and the FT estimators of fault magnitudes

Serial Number	51	52	53	54	55	56	57	75	76	77
True Values	100.0	-100.0	100.0	-100.0	100.0	0.0	0.00	100.0	0.00	0.00
LS estimators	85.93	-96.25	89.84	-98.45	97.03	-14.37	-13.15	85.41	-12.05	-11.36
FT estimators	100.92	-99.59	101.51	-99.93	102.16	-0.44	-0.38	97.63	-0.72	-0.87

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