Completing the argument

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Sorry for making the wrong definition.

I was doing the dynamics in the wrong time direction. For each clause flipped we create a tree with this clause as root and with elements of the tree to be among clauses flipped earlier. Let us examine these clauses in reverse time order starting at the time when the clause under current focus was flipped to create this tree. As stated we first put this clause as the root of the tree.

Any clause without a variable in common with any clause in the tree is ignored. Any clause with a variable in common with any clause currently in the tree is put into the tree with a directed edge from a clause in the three to the current clause. We choose an edge from the node currently in the tree with which it shares an variable and which is at greatest depth. If there are several such nodes we pick one arbitrarily.

Now consider any tree and look at the probability that it will appear during an execution of the algorithm.

Look at any node in this tree at maximal depth. As it does not have any edge leading out of it, this clause must have been falsified by the original assignment of variables and the probability of this is 2^{-k}. Remove this node from the tree and look now at any node at maximal depth in the new tree.

If the removed node was a descendent of this node then this clause must have been falsified by the original assignment together with the re-flipping of the variables joint with the first considered clause.

If the removed clause was not a descendent of the clause under scrutiny now then it must have been falsified by the original assignment. In either case the probability of this event is 2^{-k} and this event is independent of the first event considered.

Continuing inductively we see that the probability of this tree occurring during the algorithm is at most 2^{-sk}.

Combined with the fact that, as proved in class, that there are at most $m(4d)^{s-1}$ trees of size s we conclude that the expected number of trees that appear during the execution of the algorithm is at most

$$\sum_{s=1}^{\infty} m2^{-k}(4d2^{-k})^s = \frac{m2^{-k}}{1-4d2^{-k}},$$

using the assumption that $4d2^{-k} < 1$. As each flipped clause occurs as the root of one tree, the expected number of re-flipped clauses is bounded by this number and is thus of the form $O(m)$.