



KTH Computer Science
and Communication

Homework II, Theoreticians Toolkit 2009

Due on Tuesday Dec 8 at 15.15. Solutions to many homework problems, including this one, will be available on the internet, either in exactly the same formulation or with some minor perturbation. It is *not acceptable* to copy such solutions. It is hard to make strict rules on what information from the internet you may use and hence whenever in doubt contact Johan Håstad. You are, however, allowed to discuss problems in groups with up to three students, but solutions should be handed in individually.

- 1 (15p) In class Ola described how to prove that if $X = \sum_{i=1}^n X_i$ where X_i are independent and takes the values ± 1 each with probability one half then

$$\Pr[X \geq t] \leq e^{-t^2/2n}. \quad (1)$$

Prove that (1) remains true as long as the variables X_i are independent but we relax the other conditions to $E[X_i] = 0$ and that X_i belongs to the closed interval $[-1, 1]$. If you get a weaker bound on the probability in (1) of the form $e^{-at^2/2n}$ with an $a < 1$, this is sufficient for partial credit.

- 2 (15p) Let μ be a probability measure on $\{0, 1\}^k$, i.e. on k -bit strings. Show that μ is the uniform measure if and only if any non-empty exclusive-or of the bits is uniform. In other words for any nonempty $S \subseteq [k]$ when looking a

$$\bigoplus_{i \in S} X_i$$

you see a bit that is equally often 0 and 1.