Game-Theoretic Reasoning in Command and Control

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Presentation outline

- Overall research goal
- Operative level C2 modeling
- Traditional agent modeling
- Game theory
- A Bayesian game component
- An example scenario (if time permits)
- Conclusions and future work
What to expect

- The result is a *combination* of techniques rather than an improvement of existing techniques.
Overall research goal

Enhanced Situation Awareness in operative level Command and Control.

Requires reasoning under uncertainty...
Operative level Command and Control

- Uncertain environments
- Doctrine models
- Terrain models
- Bluffing opponents
- Who is the opponent?
- Is the opponent rational?
- Is there a doctrine?
Uncertain knowledge and reasoning

- Basic elements: Random variables
- Key tool: Bayesian networks
Influence diagrams

- A general mechanism for making rational decisions
- Bayesian networks extended with decision nodes (squares) and utility nodes (diamonds)
- Influence diagrams seem to be a reasonable structure for the representation of Situation Awareness
Example [Koller & Milch 2003]
Evaluating influence diagrams

1. Set the evidence variables for the current state.

2. For each possible value of the decision node;
   a) Set the decision node to that value.
   b) Calculate the posterior probabilities for the parent node.
   c) Calculate the resulting utility for the action.

3. Return the action with the highest utility.
Trying to model the C2 process in an influence diagram

- Circular relationships exist no matter how we model.
- Idea: Use all combinations of $D_1, \ldots, D_n$ to create a strategic form game.
Game theory

• Reasoning about reasoning
• Solutions are obtained in the form of (Nash) equilibria
• The game is solved for all players at the same time
• Historic events are not forgotten
Agent design perspective

• **Mechanism**
  – specifies legal actions for each agent & how the outcome is determined as a function of the agents’ strategies

• **Strategy**
  – agent’s mapping from known history to action

• A rational self-interested agent chooses its strategy in order to maximize its own expected utility given the mechanism
Nash equilibria

• A strategy profile is a *Nash equilibrium* if no player has incentive to deviate from his strategy given that others do not deviate: for every agent $i$, $u_i(s_i^*, s_{-i}) \geq u_i(s_i', s_{-i})$ for all $s_i'$

• Sometimes an agent’s best response depends on others’ strategies: a dominant strategy does not exist
Example

In a mixed strategy equilibrium, each pure strategy that occurs in the mix of agent $i$ has equal expected utility to $i$.
Friendly versus unfriendly agents

• “Friendly” environments:
  – Example: Network capacity sharing
  – Example: Sensor management
  – Agents can agree on the mechanism

• “Unfriendly” environments:
  – Opposing agents
  – Typical situation in threat prediction and situation analysis in higher level C2
  – Agents cannot agree on the game mechanism beforehand...
Bayesian games

• Idea: As we cannot agree with our opponent on what game we are playing we introduce a prior probability on what game is actually played into the model.

• Solutions/equilibria are computed by introducing an historical chance node over the possible player types.
Bayesian games cont.

- Agents may benefit from speculating about each others’
  - preferences
  - rationality
  - talents
  - capabilities …

- I.e., properties that characterizes the earlier-mentioned operative level C2 situation.
Architecture overview

$p_i(t_{i-1}, t_i)$
Computing equilibria

• Finding solutions in the form of equilibria is the foremost goal in game theory.

• Existence of an equilibrium in mixed strategies follows from Nash[1951].

• Many different types of equilibria, with different “strength”, exist.
Bayesian equilibrium

- Bayesian equilibrium [Harsanyi 1967-1968]:
  - An equilibrium is any set of mixed strategies for each type of each player, such that each type of each player is maximizing his own expected utility given that he knows his own type but does not know the other players’ types.
Complexity considerations

• Traditional method:
  – Put the game on strategic form and solve the equations
  – \textit{PSPACE}-complete

• [Stengel 1996]:
  – Sequential form (keep the game on extensive form, look at the leaves and solve in a bottom-up-fashion)
  – Computationally feasible
Problem or opportunity?

• Games being evolved from influence diagrams will be on normal form.
  – Combinatorial explosion.

• Several influence diagrams yield a structure resembling that of an extensive game.
  – Efficient algorithms exist for extensive games.

• Can these algorithms be combined?
The Härnösand scenario
Härnösand cont.

(a) 

(b)
Strategic form of the scenario

<table>
<thead>
<tr>
<th></th>
<th>$R$</th>
<th>$T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D$</td>
<td>0, 0</td>
<td>$\alpha$, $-\alpha$</td>
</tr>
<tr>
<td>$F$</td>
<td>1, -1</td>
<td>$-2\alpha$, $2\alpha$</td>
</tr>
</tbody>
</table>
Härnösand equilibrium

• Let \( q = \Pr[\text{row player chooses D}] \) and \( s = \Pr[\text{column player chooses R}] \)

• Equilibrium: expected payoff for all alternatives should be the same, i.e.,
  \[ s0 + (1-s)\alpha = s1 + (1-s)(-2\alpha) \Rightarrow s = \frac{3\alpha}{1+3\alpha} \]

• Similarly, \( q = \frac{1+2\alpha}{1+3\alpha} \)
Solution interpretation

$q(\alpha)$

$s(\alpha)$
Conclusions

• Influence diagrams provides an understandable mechanism for managing uncertainty in Command and Control.

• Game-theoretic tools provide solutions when opposing agents reason about each other.
Future work

• Further work on the algorithmic aspects regarding the connection between the influence diagram and the game component.

• Development of efficient algorithms for computing equilibria.

• Implementation.
End of presentation

1. Questions?
2. Coffee!

Decision Support Group:
http://www.nada.kth.se/theory/dsg/