

# Learning Numerical Methods for PDEs from the Web

<http://pde.fusion.kth.se>

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## Course shared between universities (1997-)

- 3 weeks introduction applied math & engineering
- 16 under-/graduates from Stockholm & Göteborg
- problem based learning with numerical experiments
- changed teaching → learning centered approach

## Students from abroad via the Web (2000-)

- life-long learning for individuals and professionals
- flexible working scheme (anytime, anywhere)

## Join this summer's course (Aug-Sep, 2001)

- invited lecturers video-conferenced abroad
- ETH Lausanne, MIT Cambridge, your university?

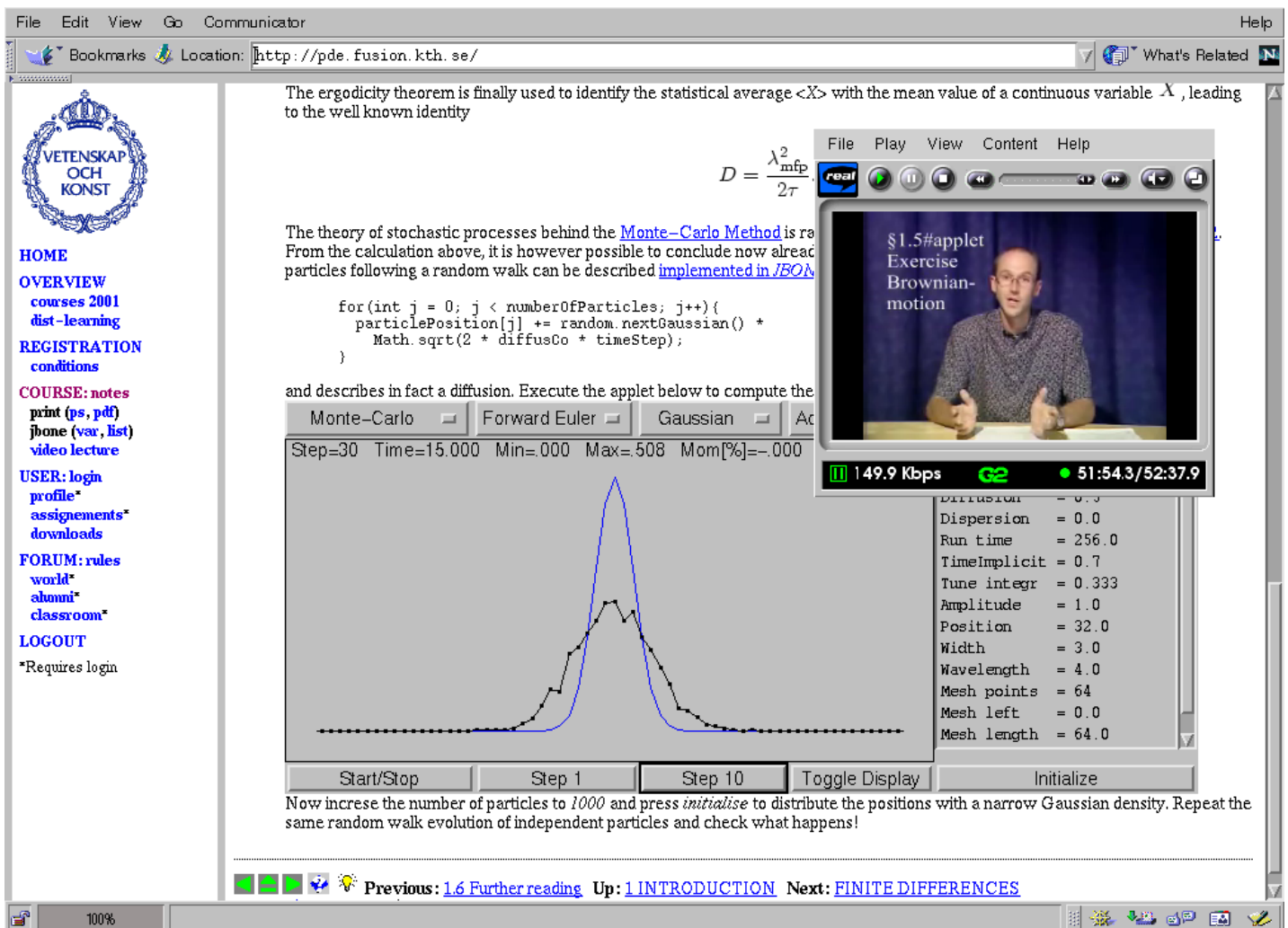
# Teaching: a Learning Centered Approach

## Passive: course notes & RealVideo recordings

- re-play, pause and skip the lessons on the Web
- conventional reading, watching & listening

## Active: experiments test your understanding

- edit the parameters in a Java-powered document
- exploit animation and interactivity



The ergodicity theorem is finally used to identify the statistical average  $\langle X \rangle$  with the mean value of a continuous variable  $X$ , leading to the well known identity

$$D = \frac{\lambda^2_{\text{mfp}}}{2\tau}$$

The theory of stochastic processes behind the [Monte-Carlo Method](#) is rather complex. From the calculation above, it is however possible to conclude now already that particles following a random walk can be described [implemented in JBOA](#)

```
for(int j = 0; j < numberOfParticles; j++){
  particlePosition[j] += random.nextGaussian() *
    Math.sqrt(2 * diffusCo * timeStep);
}
```

and describes in fact a diffusion. Execute the applet below to compute the

Monte-Carlo  Forward Euler  Gaussian  Ac

Step=30 Time=15.000 Min= 000 Max= 508 Mom[%]=-.000

Start/Stop Step 1 Step 10 Toggle Display Initialize

Now increase the number of particles to 1000 and press *initialise* to distribute the positions with a narrow Gaussian density. Repeat the same random walk evolution of independent particles and check what happens!

Previous: [1.6 Further reading](#) Up: [1 INTRODUCTION](#) Next: [FINITE DIFFERENCES](#)

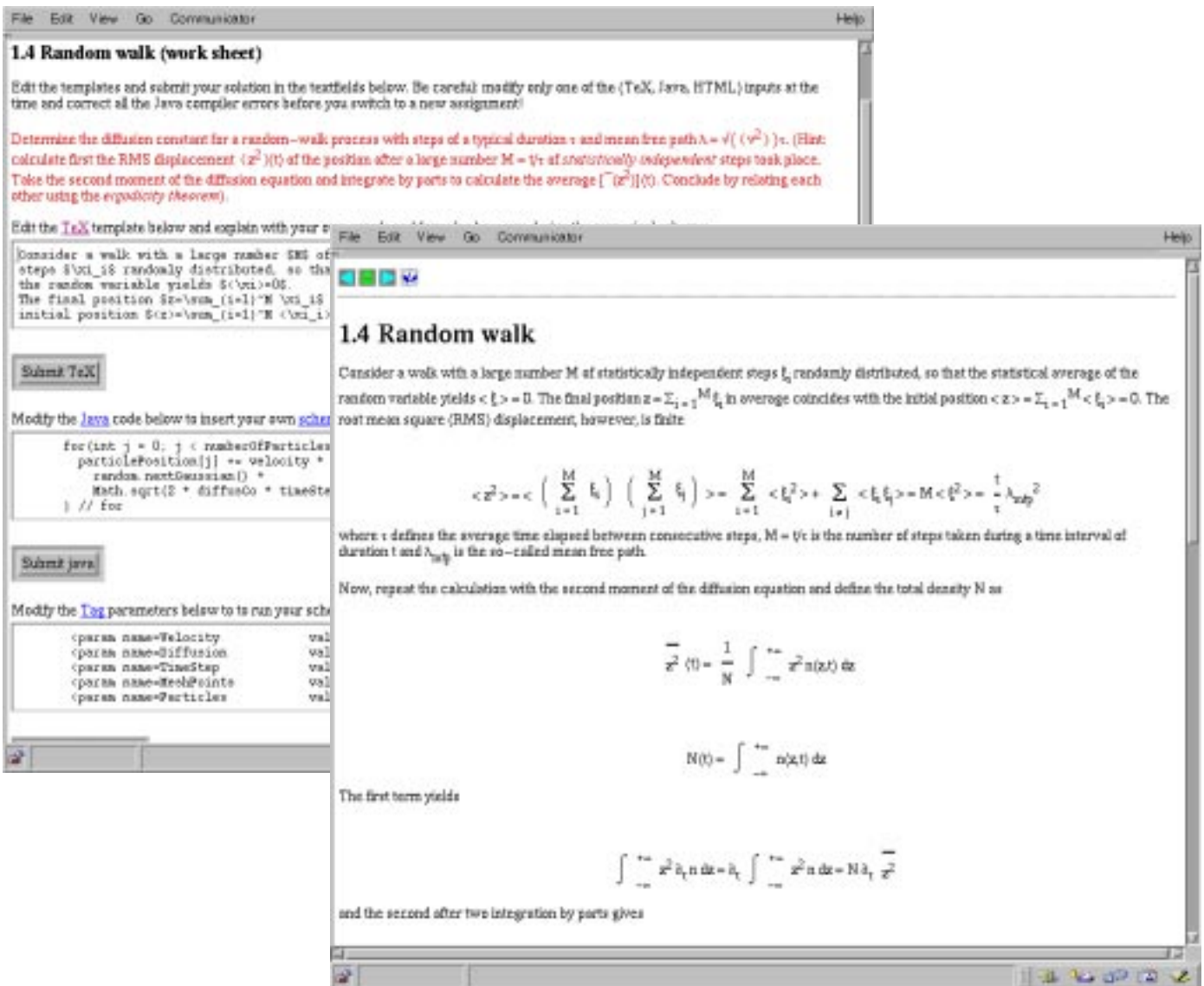
## Topics include for example:

- diffusion, Schrödinger, Black-Scholes, Burger, KdV
- finite-differences/-elements, Fourier, Monte-Carlo

# Assignments, Corrections and Support

Work out and submit exercises electronically:

- equations (T<sub>E</sub>X) & programs (Java) automatically compiled and displayed in a regular Web browser
- technology acquired in less than 2h with templates



**1.4 Random walk (work sheet)**

Edit the templates and submit your solution in the textfields below. Be careful: modify only one of the (T<sub>E</sub>X, Java, HTML) inputs at the time and correct all the Java compiler errors before you switch to a new assignment!

Determine the diffusion constant for a random-walk process with steps of a typical duration  $\tau$  and mean free path  $\lambda = \sqrt{\langle v^2 \rangle} \tau$ . (Hint: calculate first the RMS displacement  $\langle z^2 \rangle(t)$  of the position after a large number  $M = t/\tau$  of statistically independent steps take place. Take the second moment of the diffusion equation and integrate by parts to calculate the average  $\langle z^2 \rangle(t)$ . Conclude by relating each other using the ergodicity theorem).

Edit the T<sub>E</sub>X template below and explain with your own words:

Consider a walk with a large number  $M$  of steps  $\xi_i$  randomly distributed, so that the random variable yields  $\langle \xi_i \rangle = 0$ . The final position  $z = \sum_{i=1}^M \xi_i$  in average coincides with the initial position  $\langle z \rangle = \sum_{i=1}^M \langle \xi_i \rangle = 0$ . The root mean square (RMS) displacement, however, is finite:

$$\langle z^2 \rangle = \left\langle \left( \sum_{i=1}^M \xi_i \right)^2 \right\rangle = \sum_{i=1}^M \langle \xi_i^2 \rangle + \sum_{i \neq j} \langle \xi_i \xi_j \rangle = M \langle \xi^2 \rangle = \frac{1}{\tau} \lambda_{\text{step}}^2 t$$

where  $\tau$  defines the average time elapsed between consecutive steps,  $M = t/\tau$  is the number of steps taken during a time interval of duration  $t$  and  $\lambda_{\text{step}}$  is the so-called mean free path.

Now, repeat the calculation with the second moment of the diffusion equation and define the total density  $N$  as:

$$\overline{z^2}(t) = \frac{1}{N} \int_{-\infty}^{+\infty} z^2 n(z,t) dz$$

$$N(t) = \int_{-\infty}^{+\infty} n(z,t) dz$$

The first term yields:

$$\int_{-\infty}^{+\infty} z^2 \partial_t n dz = \partial_t \int_{-\infty}^{+\infty} z^2 n dz = N \partial_t \overline{z^2}$$

and the second after two integration by parts gives:

Forum discussions and correction by humans:

- quick answers 24h/day, advice usually competent
- explain concepts in the students' own language
- corrections from a pool of teachers abroad

# Experience and Conclusion

## Video-conference and technology:

- changed lecture → Q&A, quiz, illustrations
- organized discussion peers → classrooms → teacher



- open source / free software (Java, PHP, SQL)

## Evaluation:

- students love it and acquire a working knowledge
- broaden the horizon in a multi-cultural experience
- technology is enhancing the human interaction!

## Join our international teams:

- as student, teacher (local respondent) or school