A Comprehensive Study of Control Design for an Autonomous Helicopter

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Abstract

In this paper, we compare three different control methodologies for helicopter autopilot design: linear robust multi-variable control, fuzzy logic control with evolutionary tuning, and nonlinear tracking control. The control design is based on nonlinear dynamic equations with a simplified thrust-torque generation model valid for hovering and low velocity flight. We verify the controller performance in various simulated manœuvres.

1 Introduction

Unmanned autonomous aerial vehicles, UAV, are indispensable for various applications where human intervention is considered difficult or dangerous. A helicopter can operate in different flight modes, such as vertical take-off/landing, hovering, longitudinal/lateral flight, pirouette, and bank-to-turn. Due to their versatility in maneuverability, helicopters are capable to fly in and out of restricted areas and hover efficiently for long periods of time. These characteristics make helicopters invaluable for terrain surveying, surveillance and clean-up of hazardous waste sites.

BErkeley AeRobot team, $BEAR$, has built an autonomous helicopter and is currently developing a planning and control system based on hybrid system theory [1]. A Flight Vehicle Management System (FVMS) is responsible for resolving conflicts among air vehicles, planning of the flight path, generating a proper sequence of flight modes, calculating a feasible trajectory and regulating the helicopter along the nominal trajectory. The design and implementation of a stable and reliable autopilot constitutes a first step in the conception of the hybrid FVMS. In this paper, we present three different control design methodologies: robust control, fuzzy logic control, and feedback linearization. In order to cover a wide range of flight envelopes, the FVMS employs the type of controller that is most suitable for the current flight conditions. We compare and discuss the performances of different controllers obtained in benchmark simulations of typical flight manœuvres under external disturbances and system parameter uncertainties.

2 Nonlinear Simulation Model

We based the control system design on a nonlinear model of the helicopter valid in the hover and low velocity regime. A number of researchers [5, 9] have proposed nonlinear models for the aerodynamics of the main rotor and the tail rotor in hover or in forward flight. The equations of motion are obtained by equating the sum of force and moment terms in each direction to the time derivatives of the linear and angular momentum. We treat the helicopter as the lumped parameter system that consists of main rotor, tail rotor, fuselage and horizontal and vertical stabilizers. In near-hover condition, the effect of fuselage and stabilizers are neglected and the forces and moments generated by the main rotor and tail rotor are substituted into the equation of motion described in the body-coordinate system. The helicopter is controlled by four inputs: main rotor collective θ_M, longitudinal B_1 and lateral A_1 cyclic pitch, and tail rotor collective pitch θ_T . Servo actuators are linked to these control surfaces and are modeled by first-order transfer functions. A separate engine governor regulates the throttle in order to maintain a constant rotor speed. The overall model is implemented in MATLAB Simulink environment for the design and evaluation of the controllers introduced in the following chapters.

3 Linear Robust Multivariable Control Design

Owing to the uncertain and strongly coupled helicopter dynamics under severe disturbance, MIMO robust control methods have been proposed for the helicopter control[6, 10]. For the design of linear controller for hovering, μ-synthesis control theory is chosen because of its advantages to 1) quantify the uncertainty and unmodeled dynamics present in the target system, 2) model the noise characteristics of the sensor system, and 3) specify the performance objective in quantitative manner.

Problem Definition.
Find an internally stabilizing controller K(s) such that for all perturbations Δ_{pert} representing the uncertain helicopter dynamics, the closed-loop system is stable and satisfies

$$\|T\|_\infty = \|F_L[F_U(P, \Delta_{pert}), K]\|_\infty \leq 1 \qquad (1)$$

where

$$\Delta_{pert} = \{diag[\delta_1 I_{r1}, ..., \delta_S I_{rS}, \Delta_{S+1}, ..., \Delta_{S+F}] \\ : \delta_i \in \mathbf{C}, \Delta_{S+j} \in \mathbf{C}^{m_j \times m_j}, 1 \leq i \leq S, 1 \leq j \leq F\}$$

$P(s)$ is the generalized plant, which includes the helicopter linear model and weighting matrix blocks for handling quality, sensor noise model and performance specifications. The structure of the generalized plant should be carefully designed so that the controller $K(s)$ may satisfy all of the robust stability and robust performance requirements. In Fig. 1, the interconnection diagram is shown for the helicopter control problem proposed in this section. The block G contains the nominal linear time-invariant helicopter dynamics model at hover with the approximated servo dynamics. The linear system model of the helicopter in hover is obtained by linearizing the nonlinear aerodynamic model of the helicopter in hover[9]. The resulting 8-state rigid-body equation is represented as linear differential equation in Eq.2.

$$\dot{x} = A_s x + B_s u \qquad (2)$$

where $x = [v_x^b \ v_y^b \ v_z^b \ \phi \ p \ \theta \ q \ r\ ]^T$, $u = [\theta_M \ \theta_T \ A_1 \ B_1]^T$. The first order-approximate servo dynamics is used to augment at the input side of Eq. (2). Then the system equation is normalized using the maximum allowable values of state variables and control inputs in Eq. 2.

The uncertainty or unmodeled dynamics of the helicopter system equation may be categorized as: 1) poorly identified or time-varying aerodynamics or inertial quantities, 2) unmodeled higher order dynamics such as rotor flapping dynamics or servo actuator dynamics, 3) nonlinear effects of governing equations. All of these may perturb the resulting closed-loop linear/nonlinear system out of the region of stability, so the controller should be designed to robust to those effects. Among those, special attention has been given to the variation of the rotor speed and the mass of the whole vehicle, which has significant impact on entire system dynamics. Two linear system equations with the combinations of the extreme values of rotor RPM and mass are found and modeled as the matrix perturbation as shown in Eq. 3.

$$\dot{x} = (A_s + \delta_1 \Delta A)x + (B_s + \delta_2 \Delta B)u \qquad (3)$$

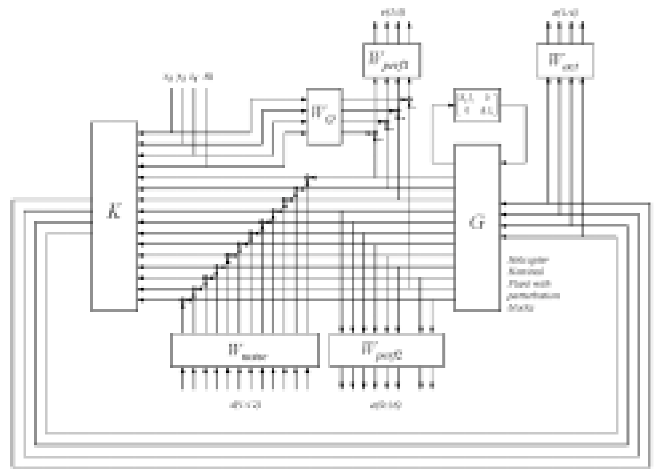

Figure 1: The interconnection diagram of the control problem

The helicopter is equipped with an INS/GPS system, which outputs twelve translational and angular measurements, i.e., (x^p, y^p, z^p, $v_x{}^b$, $v_y{}^b$, $v_z{}^b$, ϕ, θ, ψ, p, q, r). The noise characteristics are captured in the error weighting block W_{noise} using first order transfer functions with appropriate gains representing the noise level as the function of the over frequency range.

The performance specification blocks, W_{perf1} and W_{perf2}, have two important purposes. The former weights the deviation of the system output x, y, z and the heading ψ from the the desired output profiles generated by the handling quality function block T_{ideal}. It should be noted that it is necessary and sufficient to specify only those four variables to guide the helicopter following the desired trajectory. Other state variables such as roll ϕ and pitch θ and linear and angular velocity outputs are penalized by the weighting function block W_{perf2} so that the nonlinear mechanical system does not deviate from the region where the linearity assumption stands valid. The weighting function W_{act} penalize the normalized control output from $K(s)$ in the region $[-1, 1]$ so that no mechanical saturation occurs in each channel.

The controller is obtained by the algorithm known as *D-K iteration* using μ-Analysis and Synthesis Toolbox from MathWorks, Inc. The synthesized controller satisfying $\mu(F_L(P(s), K(s))) \leq 1$ for every perturbation by the predefined rotor speed and mass variations is 62nd order, which is reduced to 31st order using coprime factorization method without degrading the overall system stability and performance seriously. The resulted controller is discretized using Bilinear transform for the implementation in digital flight computer and demonstrated almost identical performance to the original full order continuous time system under various situations in the simulation.

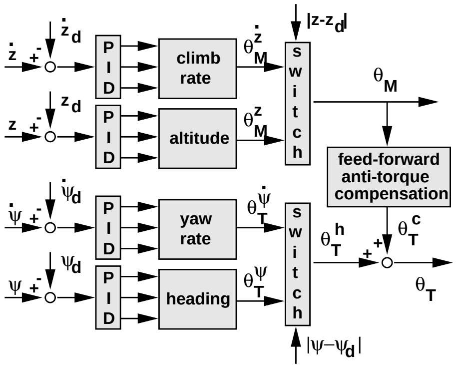

Figure 2: Fuzzy controller architecture for collective and tail rotor pitch

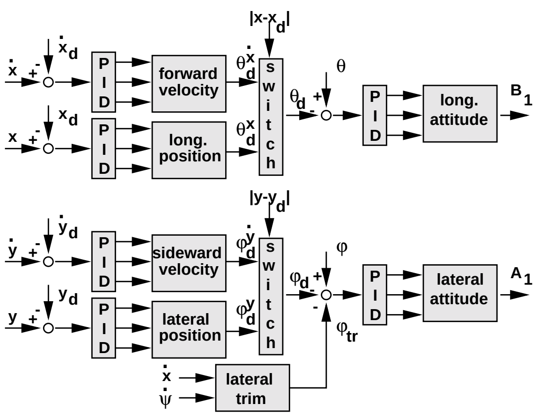

Figure 3: Fuzzy controller architecture for longitudinal and lateral cyclic pitch

4 Fuzzy Logic Control Design with Evolutionary Tuning

Among many other applications, fuzzy logic control has been applied to control an intelligent unmanned helicopter [8]. A fuzzy logic controller is a knowledge-based system characterized by a set of linguistic variables and fuzzy if-then rules. Fuzzy rules relate an input state that matches the antecedent to a control action in the consequence. Takagi, Sugeno and Kang proposed an inference scheme in which the conclusion of a rule is computed as a linear combination of the crisp inputs. Learning techniques are applicable to TSK fuzzy controllers, by adapting the gain factors of the linear control rules.

The helicopter autopilot is composed of four separate modules, which correspond to the control actuators collective pitch θ_M, tail rotor pitch θ_T , longitudinal B_1 and lateral A_1 cyclic pitch. The collective pitch control block attempts to follow a commanded altitude z_d in vertical climb and descent (Fig. 2). The heading control block governs the yaw motion during turns and compensates the anti-torque generated by the main rotor in order to maintain a desired heading ψ_d. The longitudinal and lateral block regulate the horizontal motion (Fig. 3). The cascaded architecture of the longitudinal and lateral module implicitly guarantees a stable pitch θ and roll φ attitude. Each module can either control the respective output variable or its first derivative, *i.e.*longitudinal position control for hovering or cruise control in forward flight. A fuzzy switch enables a smooth transition between both control modes. For each individual fuzzy controller, we manually derived PID gain factors, that stabilize the helicopter in the near-hover regime.

Evolutionary algorithms constitute a class of search and optimization methods, which imitate the principles of natural evolution. Their principal mode of operation is based on the same generic concepts, a population of competing candidate solutions, random combination and alteration of potentially useful structures to generate new solutions and a selection mechanism to increase the proportion of better solutions. A Genetic-Fuzzy system employs an evolutionary algorithm in order to automate the knowledge acquisition step in fuzzy control design. Phillips et al. [2] proposed a genetic algorithm to learn fuzzy logic controllers for helicopter flight. Our approach employs an evolution strategy that operates on vectors of real numbers which correspond to the gain factors c_i in the conclusion part of fuzzy rules. The population is initialized with the parameters of the hand designed fuzzy rules. The coding scheme, the genetic operators and the adaptation mechanism are described in more detail in [3].

The performance of a fuzzy controller is evaluated in a simulation of the helicopter. The designer specifies his design goals by means of a scalar objective function. In order to achieve a robust control behavior the controllers are tested under uncertainties of system parameters, a range of initial conditions and external disturbances. In the following, the altitude controller will serve as an example to illustrate the tuning process.

The altitude controller employs five PID-type rules that correspond to different errors in altitude e_z. The TSK rule for positive big (PB) errors has the form:

$$\text{if } e_z \text{ is PB then } \theta_M = c_0 + c_1 e_z + c_2 \int e_z + c_3 \dot{e_z}$$

The linear control law in the conclusion part of each rule contains the three gain factors $c_{1,2,3}$ and a constant offset c_0, which are subject to optimization by the evolution strategy.

The design objective is a fast and accurate response to changes in commanded altitude for a range of $\pm 2m$. Climb and descent manœuvres with a larger altitude change are accomplished by first using the climb rate controller to return the helicopter to the vicinity of the new altitude.

A candidate altitude controller is evaluated in simulated climb manœuvres for various payloads. The objective function that evaluates the performance of a controller includes a combination of multiple criteria, the time to reach the setpoint t_z, the amplitude of the

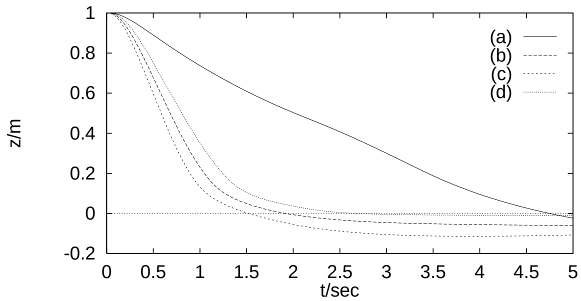

Figure 4: Vertical climb of 1m for the original (a) and optimized fuzzy altitude controller for the nominal weight (b), and $-1kg$ (c), $+1kg$ (d) payload.

overshoot a_z, the steady-state error e_z and the amplitude of final oscillations o_z. For each criteria the designer specifies a desired performance t_z^d, a_z^d, e_z^d and o_z^d. The performance index for the reaching time t_z is defined by

$$\begin{aligned} F_{t_z} &= 1 & : t_z < t_z^d \\ &= 2 - t_z/t_z^d & : t_z^d < t_z < 2t_z^d \\ &= 0 & : t_z > 2t_z^d \end{aligned} \tag{4}$$

The other indices F_{a_z}, F_{e_z} and F_{o_z} are defined in a similar fashion. The overall quality of a controller is the product of the individual performance indices.

$$F = F_{t_z} F_{a_z} F_{e_z} F_{o_z} \tag{5}$$

Fig.4 compares the performance of the original manually designed controller with the fuzzy rules tuned by the evolution strategy. The optimized controller shows an improved transient behavior and demonstrates its robustness towards a 10% change in helicopter weight.

The lateral and longitudinal modules are optimized in a similar fashion, taking into account additional performance indices that limit the roll and pitch motion. Our method allows the designer to specify his performance goals in the time domain by means of desired performance indices and releases her from a tedious tuning and optimization procedure.

5 Nonlinear Tracking Control Design

Input-output linearization[7] has been applied in a number of practical applications in solving output tracking problems of nonlinear dynamical systems including VTOL, STOL and fighter aircraft [4, 11, 13].

However, there is a large class of physical systems which do not satisfy the restrictive conditions for input-output linearization. In particular, the input-output linearization can only be applied to minimum phase nonlinear systems. If the system is non-minimum phase, direct apply the method results in high gain controllers. To avoid generating large feed-forward inputs, approximate input-output linearization [4] has been proposed. In [12], with positions and heading chosen as outputs, the helicopter model with rigid body motion and force and moment generation process is shown to be non-minimum phase. By neglecting the coupling between rolling(pitching) moments and lateral(longitudinal) accelerations, the approximated system with dynamic decoupling is linearizable without zero dynamics. The system is said to be slightly non-minimum phase, since the true system is non-minimum phase but the approximate system is minimum phase.

Based on approximate linearization, an output tracking controller is designed. In the following, the notation follow the ones as defined in [12]. The basic idea of approximate linearization in this paper is based on the assumption that a_{1s},b_{1s}, $\bar{T}_T/\bar{T}_M$ are small, so that we can neglect the coupling effects among the forces and moments:

$$\ddot{\bar{P}}_m = R \begin{bmatrix} 0 \\ 0 \\ -\bar{T}_M \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \tag{6}$$

$$\dot{\Theta} = \Psi\omega \tag{7}$$

$$\dot{\omega} = \mathcal{I}^{-1}(\tau^b - \omega^b \times \mathcal{I}\omega^b) \tag{8}$$

where $\bar{P} = P/g$, $\bar{T}_M = T_M/mg$ and $\bar{T}_T = T_T/mg$. The input to the system is $w = [\dot{\bar{T}}_M \; \dot{\bar{T}}_T \; \dot{a}_{1s} \; \dot{b}_{1s}]^T$. Define the state vector of the model system as $x_m = [\bar{P}_m \; \dot{\bar{P}}_m \; \Theta \; \omega \; w] \in \mathbb{R}^{16}$. Then, we choose $\{p_{xm}, p_{ym}, p_{zm}, \psi\}$ as outputs, and apply input-output linearization with dynamic decoupling algorithm. The extended system is in the following form:

$$\begin{bmatrix} p_{xm}^{(5)} \\ p_{ym}^{(5)} \\ p_{zm}^{(5)} \\ \psi^{(3)} \end{bmatrix} = b + A \begin{bmatrix} \ddot{w}_1 \\ w_2 \\ w_3 \\ w_4 \end{bmatrix}$$

of which the vector relative degree is $\{5, 5, 5, 3\}$. In above, the linearized model system does not contain any unobservable (zero) dynamics and hence is minimum phase. Hence, the system is slightly non-minimum phase. Next, the feedback control law $[\ddot{w}_1 \; w_2 \; w_3 \; w_4]^T = A^{-1}[-b + v]$ yields the linear closed loop system $p_{xm}^{(5)} = v_1$, $p_{ym}^{(5)} = v_2$, $p_{zm}^{(5)} = v_3$, $\psi^{(3)} = v_4$, which is decoupled. Thus, control objectives such as model matching, pole placement and tracking can be easily accommodated by choosing a suitable input, v.

To cope with the rotary wing dynamics and actuator dynamics, we define the state vector and input of the combined system as $x_a = [\theta_M \; \theta_T \; B \; A]^T$, $u_a = [u_M \; u_T \; u_B \; u_A]^T$ The dynamics can then be described as $y_a = [T_M \; T_T \; a_{1s} \; b_{1s}]^T = h^a(x_a, x_m)^T$, $\dot{x}_a = A^a x_a + b^a u_a = f^a + \sum_{i=1}^4 g_i^a u_{ai}$. Next, we take derivatives of each output y_{ai} to match with w_i for $i = 1, \ldots, 4$ and we have

$$T_M^{(3)} = \triangle_1 + L_{f^a}^2 L_{g_1^a} h_1^a \ddot{u}_M \tag{9}$$

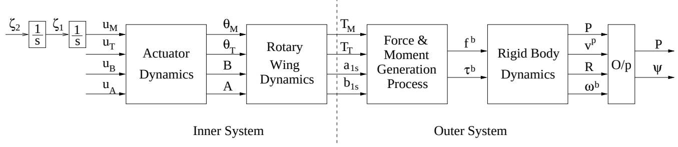

Figure 5: The approximate, dynamically extended helicopter model

$$\dot{T}_T = \triangle_2 + L_{g_2^a} h_2^a u_T \qquad (10)$$

$$\dot{a}_{1s} = \triangle_3 + L_{g_3^a} h_3^a u_B \qquad (11)$$

$$\dot{b}_{1s} = \triangle_4 + L_{g_4^a} h_4^a u_A \qquad (12)$$

Note that two extra integrators are introduced to input u_M. It can be easily verified that $L_{f^a}^2 L_{g_1^a} h_1^a \neq 0$ and $L_{g_i^a} h_i^a \neq 0$ for $i = 2, 3, 4$. Thus, the complete input-output control for the system would be

$$\begin{bmatrix} \ddot{u}_M \\ u_T \\ u_B \\ u_A \end{bmatrix} = M^{-1} \left[- \begin{bmatrix} \triangle_1 \\ \triangle_2 \\ \triangle_3 \\ \triangle_4 \end{bmatrix} + A^{-1}[-b + v] \right] \qquad (13)$$

where $M = diag\{L_{f^a}^2 L_{g_1^a} h_1^a, L_{g_2^a} h_2^a, L_{g_3^a} h_3^a, L_{g_4^a} h_4^a\}$ For output tracking control purpose, we apply

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} y_{d1}^{(\gamma_1+1)} - \alpha_1^1 e_1^{(\gamma_1)} - \cdots - \alpha_{\gamma_1+1}^1 e_1 \\ \vdots \\ y_{d4}^{(\gamma_4+1)} - \alpha_1^4 e_4^{(\gamma_4)} - \cdots - \alpha_{\gamma_4+1}^1 e_4 \end{bmatrix} \qquad (14)$$

where $e_i^{(j)} = \xi_{j+1}^i - y_{di}^{(j)}$ for $j = 0, \ldots, \gamma_i$ and $i = 1, \ldots, 4$ and the polynomials $s^{\gamma_i+1} + \alpha_1^i s^{\gamma_i} + \cdots + \alpha_{\gamma_i+1}^i$ chosen Hurwitz. Given output trajectories with sufficiently small derivatives and the derived output tracking controller, the states of the the closed loop system and the tracking error can be proved to be bounded.

A block diagram showing the structure of the approximate, extended system is presented in Fig. 5. As stated in [12], the extended system is approximately differentially flat since the system can be linearized by approximate feedback. Thus, one can generate approximate state and nominal input trajectory for the true system from the output trajectory for state tracking purpose. Furthermore, the approximate, extended system can also be approximately linearized without undesirable internal dynamics by choosing other output sets in order to be operated in different flight modes.

6 Simulation

In this section we compare the performance of the controllers designed by using linear robust multi-variable control, fuzzy logic control with evolutionary tuning, and nonlinear tracking control in regard to disturbance rejection, uncertainties in system parameter and tracking accuracy. The control behavior is evaluated for two typical scenarios of helicopter flight, a vertical climb (Fig. 7) and a simultaneous longitudinal and lateral motion (Fig. 6). The transient and steady-state responses are observed for the nominal helicopter model without disturbance. We introduce model uncertainty by increasing the helicopter weight by 10% in the case of vertical climb (Fig. 7). For the horizontal manœuvre the helicopter is exposed to a constant wind gust of $14m/s$ in north-east direction (Fig. 6).

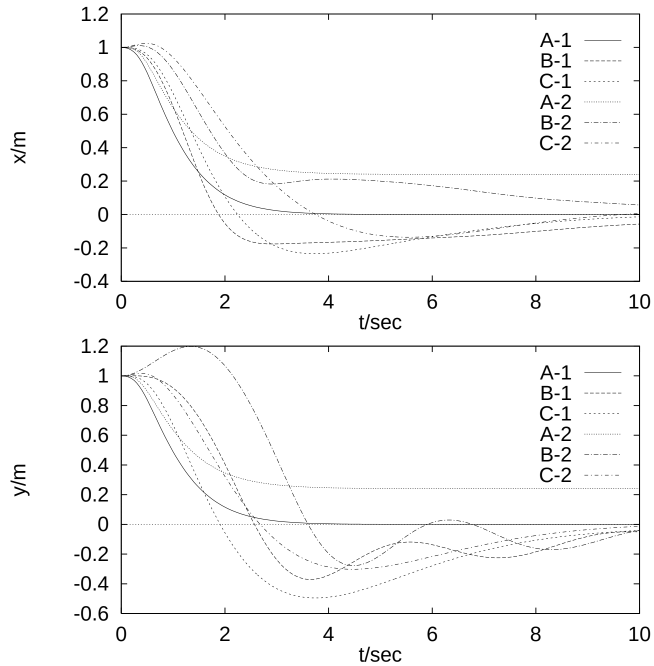

Figure 6: Simultaneous longitudinal (upper) and lateral (lower) manœuvre using nonlinear (A), fuzzy (B) and robust (C) control for the nominal (1) model and in the presence of a constant wind gust (2).

The nonlinear controller demonstrates accurate tracking performance for the nominal system but shows a steady-state error in case of external disturbances. It also turns out that the nonlinear controller is sensitive to model disparities, such as changes in the payload or to the aerodynamic thrust-torque model. The tracking error can be reduced by placing the poles further away from the origin in the left-half plane, at the cost of a higher control input magnitude.

While the robust controller exhibits a moderate transient response it shows the robustness to the disturbance and model uncertainty. The robust controller is designed to trade off between the robustness and the tracking performance. In our design, a greater emphasis is put on the robust stability in order to guarantee safe helicopter operation in the presence of severe disturbance such as ground effects and side winds.

The fuzzy controller achieved the performance criteria specified by the objective function for the nominal as well as the disturbed models. The fuzzy controller is

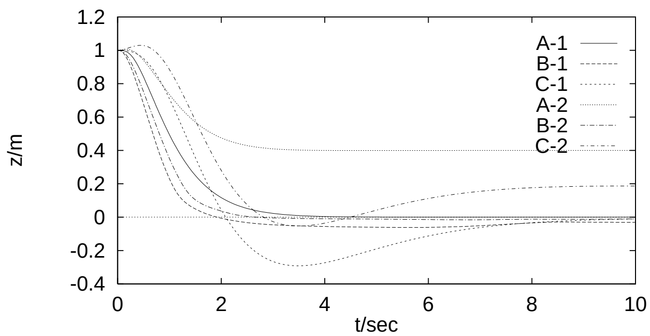

Figure 7: Vertical climb manœuvre using nonlinear (A), fuzzy (B) and robust (C) control for the nominal (1) model and for a 10% increase in helicopter weight (2)

able to minimize the steady-state error because it utilizes the integral of error for control.

7 Conclusion

This paper presented three different control methodologies for helicopter autopilot design. Our simulation results show that the robust and fuzzy controller are capable of handling uncertainties and disturbances. Nevertheless, their operating regime is limited to near-hover conditions. On the other hand nonlinear control covers a substantially wider range of flight envelopes, but requires accurate knowledge about the system. To accommodate this insufficiency of pure feedback linearization, the pole-placement control scheme will be replaced by a robust controller based on a perturbed linearized system. The FVMS selects the proper type of controller depending on the flight conditions and commanded behavior. In the near future, the controllers will be implemented and verified under real flight situations on the UC Berkeley UAV.

8 Acknowledgment

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