Pedagogical 2D Panel Methods

A panel method code written in Matlab is used to compute 2D inviscid incompressible irrotational flow solutions. An overview of the program facilities is first presented.

* Three different kinds of **geometries** are implemented :
- Ellipse with prescribed axis ratio,
- NACA 4 digits airfoil library (see reference [1]),
- General airfoil library.

In the two first cases, the shape of the object is defined analytically. The user simply has to enter the proper information, ellipse axis ratio or Naca 4 digits.

In the last case, the airfoils are defined in data files using a number of discrete points. Around 70 airfoils taken from well known shape libraries are already recorded and ready to use. New data files can be added to the library with any airfoil coordinates except that the airfoil trailing edge must be sharp.

* Three different kinds of **singularities** are implemented (theory can be found in reference [2]).

Type	Variation	Boundary Condition	Potential of point singularity	Velocity of point singularity (singularity located at (0,0))
Source	Constant	Neumann	$\frac{\sigma}{2\pi}\ln\sqrt{x^2+y^2}$	$\frac{\sigma}{2\pi}\left(\frac{x}{x^2+y^2};\frac{y}{x^2+y^2}\right)$
Doublet	Constant	Dirichlet	$-\frac{\mu}{2\pi}\left(\frac{y}{x^2+y^2}\right)$	$\frac{\mu}{\pi}\left(\frac{xy}{(x^2+y^2)^2};\frac{y^2-x^2}{2(x^2+y^2)^2}\right)$
Vortex	Linear	Neumann	$-\frac{\Gamma}{2\pi}\operatorname{atan}\frac{y}{x}$	$\frac{\Gamma}{2\pi}\left(\frac{y}{x^2+y^2};\frac{-x}{x^2+y^2}\right)$

Note : A vortex distribution of strength γ can be replaced by an equivalent doublet distribution of strength μ such that :

$$\gamma(x) = -\frac{d\mu(x)}{dx}$$

So, a linear vortex distribution is equivalent to a quadratic doublet distribution,

* The user may change the flow **angle of attack**, as well as the **number of panels** used in the discretization.

* The **solution** computed by the program features the C_p distribution and the aerodynamics coefficients c_l and c_m.

*** Miscellaneous :**

It is possible to zoom in and out on the 2D plot. Get information by entering “help zoom” in Matlab. It is also possible to hold on the previous plot by clicking on the button “Hold on (Cp)”. This is allowed only between solutions on the same geometry and at the same angle of attack.

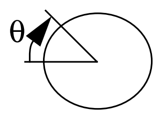
Exercises

For the 4 first questions, use the linear vortex singularity model.

1- Flow past an ellipse

1.1- Compute and plot the pressure distribution corresponding to the flow around a circular cylinder using different number of panels (for instance 8,16,32,...). Analyze the results and give the number of panels necessary to achieve convergence, say an error in minimum C_p of less than xx%.

1.2- Compare the converged C_p distribution with the theoretical cylinder solution :



$$cp = 1 - 4\sin^2\theta$$

2- NACA 4 digits library

2.1- Recall the exact meaning of the 4 digits, make a drawing and give an example.

2.2- In this question, you will study the effect of the three shape parameters thickness, camber and camber location taken separately on c_l and c_m.

Make a preliminary study to find the minimum number of panels required for convergence on a typical test case, then make sure that you use the same number of panels for all cases.

Report c_l and c_m for 3 sets of Naca airfoils : one with variable thickness, the second with variable camber and the third with variable camber location. Plot the corresponding so-called sensitivity curves c_l and c_m vs parameter. Draw a conclusion which sums up what you obtain on these curves.

2.3- Using your conclusions, solve the following optimisation problem :

max c_l(max thickness, max camber, max camber location)
subject to : $4 \le$ max thickness ≤ 20
$2 \le$ max camber ≤ 9
$2 \le$ max camber location ≤ 9

Give the optimiser and the optimum.

Hint : Do not make any computation ! *c_l is a convex function, which means that it has only one local maximum. Given the sensitivity curves from 2.2, the answer should then be a "one-liner".*

Explain why the solution to this problem is not physically relevant. Which important constraints should be added for interesting shape optimisation ?

3- General airfoil library

3.1- Play around with airfoils from the library at zero angle of attack. For three airfoils with very different solutions, report the C_p curves and aerodynamics coefficients and analyse the differences.

3.2- Influence of the geometry on the solution

- Compute the C_p on the following airfoils at zero alpha : VEZBL32, VEZCAN, andVEZWLTR. What do you observe ? Find an explanation by taking a close look at the airfoil geometry.
- Compute the C_p on the KORN airfoil at zero alpha using constant-strength doublets, use 100 panels and then 200 panels. What do you observe ? Find an explanation by taking a close look at the airfoil trailing edge.

3.3- Choose an airfoil in [1] (see the attached pages), and enter the coordinates in a file name.DAT. Use the right format for the data ! (Can be seen in any .DAT file in the library). Compute a converged solution, and compare the aerodynamic coefficients with experimental data from [1].

4- Effect of the angle of attack

Select 3 different airfoils, a thin, a medium, and a thick one. For each of them, plot a curve c_l vs alpha for values of alpha varying between -10 and +10 degrees in increments of 2 degrees. Compare the slope of the curves to the theoretical 2*pi for very thin airfoils.

5- Effect of the type of singularity

5.1- Study of source panel method : Choose a symmetric airfoil at zero angle of attack, and compute the solution using sources. Compare with the solution obtained using doublets or vortices. Then do the same but with a non-symmetric airfoil, or a symmetric airfoil at non-zero angle of attack. What do you observe ? Find an explanation based on the capabilities of source panel methods.

5.2- Comparison between constant doublets and linear vortices : Select the Naca 8812 airfoil, and compute the solution using constant doublets and then linear vortices with the same number of panels, say 50. Now, use more panels for the doublets computation, say 250, and compare the solution to the one obtained previously with 50 linear vortex panels. What do you observe ? Find an explanation based on the relation between linear vortices and constant doublets.

References

[1] I.H. Abbott and A.E. Von Doenhoff, Theory of wing sections, Dover Publications Inc, NewYork, 1959

[2] Katz and Plotkin : Low Speed Aerodynamics, From Wing Theory To Panel Methods. McGraw-Hill Inc., 1991