

Climate Thermodynamics

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1 Global Climate by Navier-Stokes Equations

Thermodynamics is a funny subject. The first time you go through it, you don't understand it at all. The second time you go through it, you think you understand it, except for one or two small points. The third time you go through it, you know you don't understand it, but by that time you are so used to it, it doesn't bother you any more. (Physicist Arnold Sommerfeld (1868-1951))

Global climate results from a thermodynamic interaction between the atmosphere and the ocean with radiative forcing from the Sun, gravitational forcing from the Earth (and the Moon) and dynamical Coriolis forcing from the rotation of the Earth. The thermodynamics is described by the *Navier-Stokes equations* (NSE) of fluid dynamics, for a variable density incompressible ocean and compressible atmosphere, expressing conservation of mass, momentum and energy.

The atmosphere transports heat energy absorbed by the Earth surface from the Sun to a top of the atmosphere TOA from where it is radiated to outer space, and thus acts as an air conditioner or heat engine [8] keeping the surface temperature constant under radiative forcing from the Sun. A basic question in climate science is the stability of this air conditioner under varying forcing, more specifically the change of surface temperature under doubled concentration of atmospheric CO_2 (from 0.028% to 0.056%), referred to as *climate sensitivity*.

The heat is transported by the atmosphere in a combination of thermodynamics (turbulent convection and phase change in evaporation/condensation) and radiation, roughly 2/3 by thermodynamics and 1/3 by radiation. The thermodynamics involves positive radiative forcing balanced by evaporation at low latitudes/altitudes from a warm ocean causing warm air to rise-expand-cool including poleward motion followed by negative radiative forcing balanced by condensation at high latitudes/altitudes causing cool air to descend-contract-warm closing a thermodynamic cycle, as indicated in Fig. 1, during polar winter.

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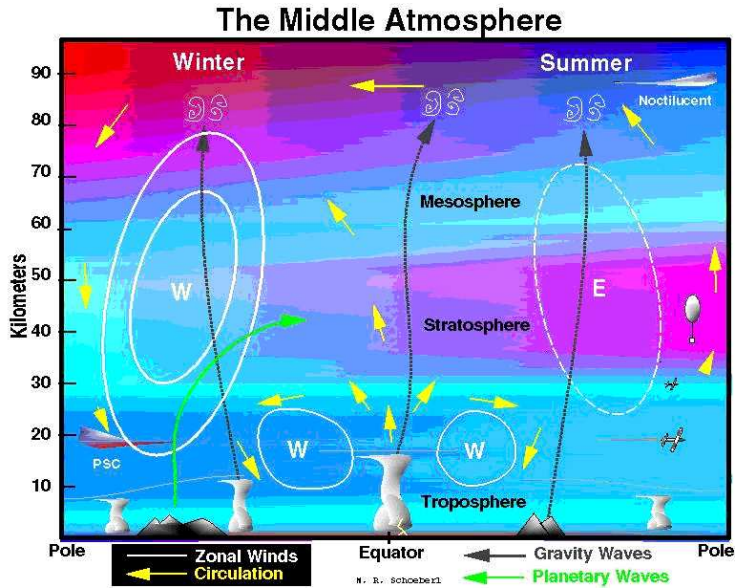


Figure 1: Thermodynamics of the atmosphere (NASA UARS Project).

2 The Illusory Greenhouse Effect

The main message to the World and its leaders from the 2007 IPCC Fourth Assessment Report (AR4) is a prediction of an alarming climate sensitivity in the range 2 – 4.5 C as a result of a so-called *greenhouse effect*.

The physics of this effect is claimed to have been identified and scientifically described by Fourier[3] (1824), Tyndall[10] (1861) and Arrhenius[1] (1896). An inspection of these sources shows a very simplistic rudimentary analysis with a only a simple model for radiation and no thermodynamics, which is the origin of the message of this article: The mathematics of the Fourier-Tyndall-Arrhenius greenhouse effect is dead, and never was alive!

However, to confuse the discussion, the “greenhouse effect” is described with a misleading double-meaning: It is both the combined total effect of the atmosphere on the Earth surface temperature including both radiation and thermodynamics, and at the same time a hypothetical radiative effect of “greenhouse gases” including CO_2 without thermodynamics. In this way the “greenhouse effect” becomes real, because it is the total effect of the atmosphere and the atmosphere undeniably has an effect, an “atmosphere effect”, while at the same time it can be linked to CO_2 acting like a powerful “greenhouse gas” capable of global warming upon a very small increase of 0.028%.

The simplest version of the “greenhouse effect” is described by Stefan-Boltzmann’s

Law $Q = \sigma T^4$ (SBL), which in differentiated form

$$dQ = \sigma 4T^3 dT = 4 \frac{Q}{T} dT \sim 4dT$$

with $Q \approx 280 \text{ W/m}^2$ and $T \approx 288 \text{ K}$, gives a climate sensitivity of about 1 C by attributing a certain fictitious additional “radiative forcing” $dQ = 4 \text{ W/m}^2$ to doubled CO_2 .

Since the total radiative forcing from the Sun is assumed to not change, the additional radiative forcing is supposed to result from a shift of the “characteristic emission level/altitude” to a higher level at lower temperature caused by less radiation escaping to space from lower levels by increasing absorption by CO_2 . In this argument the outgoing radiation is connected to a *lapse rate* (decrease of temperature with increasing altitude) supposedly being determined by thermodynamics. With lower “characteristic emission temperature” at higher altitude the whole temperature profile will have to shifted upwards thus causing warming on the ground.

This is the starting point of the climate alarmism propagated by IPCC, a basic climate sensitivity of 1 C , which then is boosted to $2 - 4.5 \text{ C}$ by various so-called (positive) “feed-backs”. The basic argument is that since Stefan-Boltzmann’s Law cannot be disputed as such, and because CO_2 has certain properties of absorption/emission of radiation (light), which can be tested in a laboratory, the starting value of 1 C is an “undeniable physical fact which cannot be disputed”. Even skeptics like Lindzen and Spencer accept it, and if even skeptics believe something, then it must be true, right?

But wait! Science does not work that way, science only obeys facts and logical mathematical arguments, the essence of the scientific method, and let us now check if the basic postulate of a “greenhouse effect” with basic climate sensitivity of 1 C can qualify as science. And climate politics without live climate science is dead politics.

3 Mathematical Climate Simulation

The language and methodology of science, in particular climate science, is mathematics: Physical laws are expressed as differential equations of the principal form $D(u) = F$, where F represents *forcing*, u represents the corresponding *system state* coupled to F through a differential operator $D(u)$ acting on u . With given forcing F , the corresponding state u can be determined by solving the differential equation $D(u) = F$. This is the essence of the scientific method. Note that the differential equation $D(u) = F$ usually describes a cause-effect relation in the sense that the system state u responds to a known given forcing F in a (stable) *forward problem*. This corresponds to putting the horse in front of the wagon, and not the other way around which is referred to as an (unstable) *inverse problem* with the state u given and F the forcing being sought.

Consider now the following approaches to model and simulate global climate:

- (A) Thermodynamics with radiative forcing (NSE with SBL forcing).
- (B) Radiation $dQ \sim 4dT$ as differentiated form of (SBL).

- (C) Radiation $dQ \sim 4dT$ combined with thermodynamic lapse rate.
- (D) Radiation $dQ \sim 4dT$ combined with thermodynamic lapse rate and feedback.

Here (A) is the (stable) forward problem described in the first section and studied below. (B) is self-referential without thermodynamics. (C-D) represent the IPCC approach as an (unstable) inverse problem of radiation with thermodynamic forcing with potentially large positive feed-backs and high climate sensitivity.

Altogether, (A) opens to a rational scientific approach as a stable forward problem, whereas the (C-D) of IPCC represents an unstable inverse problem of questionable value.

In its popular form the basic IPCC climate sensitivity of 1 C is claimed to come from a “greenhouse gas” ability of CO_2 to “trap heat”, which is supposed to convince the uneducated. In its more elaborate form intended for the educated, it is connected to a thermodynamic lapse rate and characteristic emission level, in order to account for an effect of additional radiative forcing without change of total radiative forcing. Both forms are severely simplistic and cannot count as science.

To follow (A) we must rid ourselves from the common misconception of thermodynamics expressed in the quote above by Sommerfeld, that it is beyond comprehension for mortals, in particular its 2nd Law. This is the reason why climate scientists have focussed on radiation only, as something understandable, backing away from thermodynamics as something nobody can grasp. But it is possible to give thermodynamics and the 2nd Law a fully understandable meaning as I show in [4, 5] and recall below. This insight opens to a rational approach to climate dynamics, as (A) thermodynamics with radiative forcing.

4 Lapse Rate and Global Warming/Cooling

A theory is the more impressive the greater the simplicity of its premises, the more different kinds of things it relates, and the more extended its area of applicability. Therefore the deep impression that classical thermodynamics made upon me. It is the only physical theory of universal content which I am convinced will never be overthrown, within the framework of applicability of its basic concepts. (Einstein)

The effective blackbody temperature of the Earth with atmosphere is -18 C , which can be allocated to a TOA at an altitude of 5 km at a lapse rate (temperature drop with increasing altitude) of 6.5 C/km connecting TOA to an Earth surface at 15 C with a total warming of $5 \times 6.5 = 33\text{ C}$. The lapse rate determines the surface temperature since the TOA temperature is determined to balance a basically constant insolation. What is then the main factor determining the lapse rate: Is it radiation or thermodynamics, or both?

Climate alarmism as advocated by IPCC is based on the assumption that radiation alone sets an initial lapse rate of 10 C/km , which then in reality is moderated by thermodynamics to an observed 6.5 C/km . Doubled CO_2 would then increase the initial lapse rate and with further positive thermodynamic feedback it is by IPCC predicted to an alarming climate sensitivity or global warming of $1.5 - 5\text{ C}$. Climate alarmism



Figure 2: The atmosphere maintains a constant surface temperature under increasing radiative heat forcing by increasing vaporization and turbulent convection, like a boiling pot of water on a stove.

skeptics like Richard Lindzen and Roy Spencer buy the argument of an initial rate of $10\text{ C}/\text{km}$ determined by radiation, but suggest that negative thermodynamic feedback effectively reduces climate sensitivity to a harmless 0.5 C .

We will argue that an initial lapse rate of $g = 9.81\text{ C}/\text{km}$ is instead determined by thermodynamics (and not by radiation) as an equilibrium state without heat transfer, which then in reality by thermodynamic heat transfer (turbulent convection/phase change) is decreased to the observed $6.5\text{ C}/\text{km}$, with the heat transfer balancing the radiative heat forcing. More CO_2 would then require more heat transfer by thermodynamics and thus to a further decrease of the lapse rate rather than an increase. The atmosphere would then act like a boiling pot of water which under increased heating would boil more vigorously but not get any warmer.

In short: If thermodynamics is the main mechanism of the atmosphere as an air conditioner or heat transporter, then CO_2 will not cause warming, and IPCC climate alarmism collapses.

We thus identify a basic difference between atmospheric heat transport by radiation (similar to conduction) and by thermodynamics of convection/phase change. In radiation/conduction increased heat transport couples to increased lapse rate (warming). In convection/phase change increased heat transport couples to decreased lapse rate (cooling).

5 Euler Equations for the Atmosphere

Every mathematician knows it is impossible to understand an elementary course in thermodynamics. (Mathematician V. Arnold)

The viscosity of both water and air is small, while the spatial dimensions of the

ocean and atmosphere are large, which means that the Reynolds number $Re = \frac{UL}{\nu}$ is very large ($> 10^8$), where $U > 1 \text{ m/s}$ is a typical velocity, $L > 10^3 \text{ m}$ a length scale and $\nu < 10^{-5}$ a viscosity. Global climate thus results from turbulent flow at very large Reynolds numbers effectively in the form of turbulent solutions of the *Euler equations* as described in [4].

We focus now on the atmosphere and as a model we consider the Euler equations for a compressible perfect gas occupying a volume Ω representing e.g. the troposphere, here for simplicity without Coriolis force from rotation: Find (ρ, u, T) with ρ density, u velocity and T temperature depending on x and $t > 0$, such that for $x \in \Omega$ and $t > 0$:

$$\begin{aligned} D_u \rho + \rho \nabla \cdot u &= 0, \\ D_u m + m \nabla \cdot u + \nabla p + g \rho e_3 &= 0, \\ D_u T + RT \nabla \cdot u &= q, \end{aligned} \tag{1}$$

where $m = \rho u$ is momentum, $p = R\rho T$ is pressure, $R = c_p - c_v$ with c_v and c_p specific heats under constant volume and pressure, and $D_u v = \dot{v} + u \cdot \nabla v$ is the material time derivative with respect to the velocity u with $\dot{v} = \frac{\partial v}{\partial t}$ the partial derivative with respect to time t , $e_3 = (0, 0, 1)$ is the upward direction, g gravitational acceleration and q is a heat source. For air $c_p = 1$ and $\frac{c_p}{c_v} = 1.4$. The Euler equations are complemented by initial values for ρ , m and T at $t = 0$, and the boundary condition $u \cdot n = 0$ on the boundary of Ω where n is normal to the boundary.

We assume that the heat source q adds heat energy at lower latitudes/altitudes and subtracts heat at higher latitudes/altitudes (radiation to outer space) including evaporation (subtraction of heat) at low altitudes and condensation (addition of heat) at higher altitudes.

We thus consider the full 3D (three-dimensional) Euler/Navier-Stokes equations without any simplification of the vertical flow as in 2D geostrophic flow or in hydrostatic approximation of vertical momentum balance, as a required feature of the next generation of climate models [9] not present in the current generation [2]. This is important because the heat transport involves both horizontal and vertical flow, roughly speaking ascending air at low latitudes and descending air at high latitudes, combined with high altitude poleward flow and low altitude flow towards the Equator.

6 The 1st and 2nd Laws of Thermodynamics

...no one knows what entropy is, so if you in a debate use this concept, you will always have an advantage. (von Neumann to Shannon)

We recall the 2nd Law of Thermodynamics as stated in [5]:

$$\dot{K} + \dot{P} = W - D, \quad \dot{E} = -W + D + Q, \tag{2}$$

where

$$\begin{aligned}
K(t) &= \frac{1}{2} \int_{\Omega} \rho u \cdot u(x, t) dx, & P(t) &= \int_0^t \int_{\Omega} g \rho u(x, s) dx ds, \\
E(t) &= \int_{\Omega} c_v \rho T(x, t) dx, & W(t) &= \int_{\Omega} p \nabla \cdot u(x, t) dx, \\
Q(t) &= \int_{\Omega} q(x, t) dx,
\end{aligned} \tag{3}$$

is momentary total kinetic energy $K(t)$, potential energy $P(t)$, internal energy $E(t)$ and work rate $W(t)$, and $D(t) \geq 0$ is rate of turbulent dissipation and $Q(t)$ rate of supplied heat or heat forcing. The work W is positive in expansion with $\nabla \cdot u$ positive, and negative in compression with $\nabla \cdot u$ negative (since the pressure p is positive).

Adding the two equations of the 2nd Law, we find that the change of total energy ($K + P + E$) is balanced by the heat forcing:

$$\frac{d}{dt}(K + P + E) = Q, \tag{4}$$

which can be viewed to express the 1st Law of Thermodynamics as conservation of total energy.

Thermodynamics essentially concerns transformations between heat energy E and the sum $K + P$ of kinetic and potential energies with the transfer being $\pm(W - D)$: whatever $K + P$ gains is lost by E and vice versa. The 2nd Law sets the following limits for these transformations:

- heat energy E can be transformed to kinetic/potential energy $K + P$ only under expansion with $W > 0$,
- turbulent dissipation D can transform kinetic/potential energy $K + P$ into heat energy E ,
- turbulent dissipation D cannot transform heat energy to kinetic/potential energy, because $D \geq 0$.

7 Basic Isothermal and Isentropic Solutions

As anyone who has taken a course in thermodynamics is well aware, the mathematics used in proving Clausius theorem (the 2nd Law) is of a very special kind, having only the most tenuous relation to that known to mathematicians. (Mathematician S. Brush)

We identify the following hydrostatic equilibrium base solutions, here fitted to an observed Earth surface temperature of 288 K, assuming $Q = 0$:

$$\begin{aligned}
\bar{u} &= 0, \quad \bar{T} = 288 - gx_3, \quad \bar{\rho} = \alpha(288 - gx_3)^{\frac{1}{\gamma}}, \quad \bar{p} = R\alpha(288 - gx_3)^{\frac{1}{\gamma}+1}, \\
\bar{u} &= 0, \quad \bar{T} = 288(K), \quad \bar{\rho} = \alpha \exp(-gx_3), \quad \bar{p} = R288\alpha \exp(-gx_3),
\end{aligned} \tag{5}$$

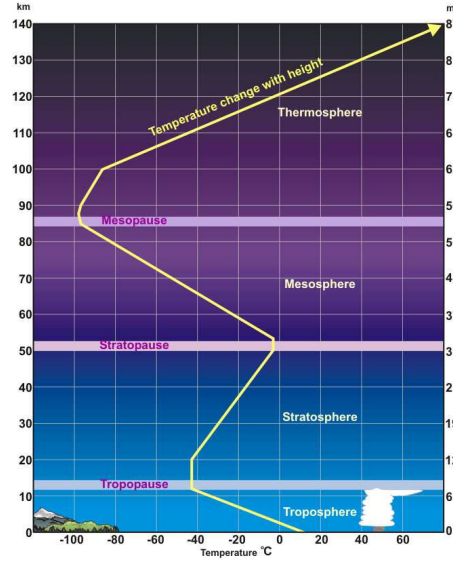


Figure 3: Temperature profile of the atmosphere, with constant lapse rate in the troposphere of $6.5\text{ C}/\text{km}$ (NOAA).

where $\gamma = \frac{R}{c_v}$ ($= 0.4$) and thus $R(\frac{1}{\gamma} + 1) = c_p = 1$, we scale x_3 in km and α denotes a positive constant to be determined by data.

The first solution is non-turbulent (or isentropic) with $D = 0$ in the 2nd Law:

$$\dot{E} + W = 0, \quad (6)$$

or in conventional notation

$$c_v dT + p dV = 0, \quad (7)$$

which combined with hydrostatic balance $\frac{\partial p}{\partial x_3} = -g\rho$ and the differentiated form $p dV + V dp = R dT$ of the gas law, gives

$$(c_v + R) \frac{\partial T}{\partial x_3} = -g. \quad (8)$$

With $c_v + R = c_p = 1000\text{ J}/\text{K kg}$ the heat capacity of dry air we obtain an isentropic *dry adiabatic lapse rate* of $10\text{ C}/\text{km}$. With the double heat capacity of saturated moist air we obtain an isentropic *moist adiabatic lapse* of $5\text{ C}/\text{km}$.

The second solution has constant temperature and exponential drop of density and pressure, and can be associated with lots of turbulent dissipation (with $D = W$) effectively equilibrating the temperature.

We summarize the properties of the above base solutions (with $Q = 0$):

- isothermal: maximal turbulent dissipation: $D = W$,

- isentropic: minimal turbulent dissipation: $D = 0$.

We find real solutions between these extreme cases, with roughly $D = \frac{W}{2}$ and $\bar{p} \sim (288 - gx_3)^5$, $\bar{p} \sim (288 - gx_3)^6$, with a quicker drop with height than for the isentropic solution with $\bar{p} \sim (288 - gx_3)^{2.5}$ and $\bar{p} \sim (288 - gx_3)^{3.5}$, or turned the other way, with a smaller lapse rate of $6.5 C/km$.

8 Basic Thermodynamics

...thermodynamics is a dismal swamp of obscurity... a prime example to show that physicists are not exempt from the madness of crowds... Clausius verbal statement of the second law makes no sense... All that remains is a Mosaic prohibition; a century of philosophers and journalists have acclaimed this commandment; a century of mathematicians have shuddered and averted their eyes from the unclean... Seven times in the past thirty years have I tried to follow the argument Clausius offers and seven times has it blanked and gravelled me. I cannot explain what I cannot understand. (Physicist C. Truesdell)

We have formulated a basic model of the atmosphere acting as an air conditioner/refrigerator by transporting heat energy from the Earth surface to the top of the atmosphere in a thermodynamic cyclic process with radiation/gravitation forcing, consisting of

- ascending/expanding/cooling air heated by low altitude/latitude radiative forcing,
- descending/compressing/warming air cooled by high altitude/latitude outgoing radiation,

combined with low altitude evaporation and high altitude condensation.

The model is compatible with observation and suggests that the lapse rate/surface temperature is mainly determined by thermodynamics and not by radiation.

The thermodynamics of a standard refrigerator requires a compressor, which in the case on an atmosphere is taken over by gravitation causing compression of descending air.

9 Basic Data

You can fool all the people some time, and some of the people all the time, but you cannot fool all people all the time. (Abraham Lincoln)

We collect the following observed data, for the first half of the above cycle:

- average upward velocity = $0.01 m/s$,
- average density = $0.6 kg/m^3$,
- average altitude of TOA = $5000 m$,
- $c_p = 1000 J/K kg$

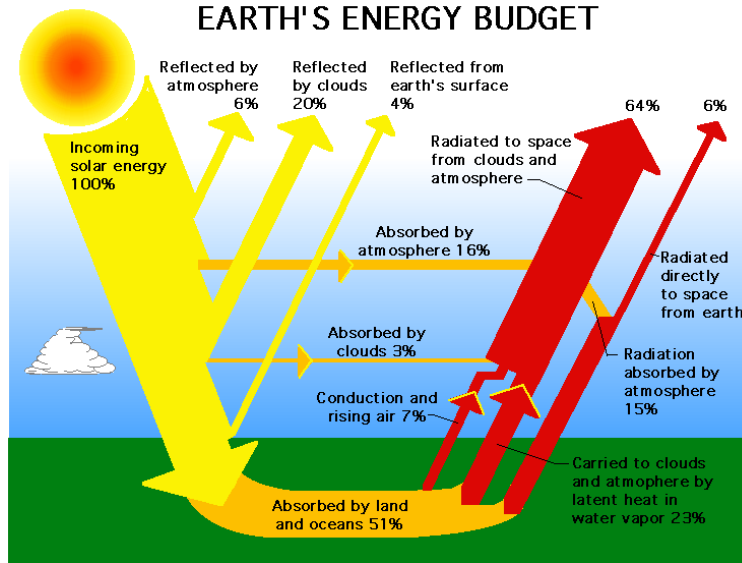


Figure 4: Earth energy budget (NASA Atmospheric Science Data Center).

- $Q \approx 180 \text{ W/m}^2$ absorbed by the Earth surface with 60 W allocated to radiation, and 120 W to thermodynamics with 100 W to evaporation and 20 W to convection.
- observed lapse rate $\approx -6.5 \text{ C/km}$,
- evaporation $\approx 4 \text{ cm/day}$,
- heat of vaporization of water 2200 kJ/kg ,
- turbulent dissipation rate: 0.002 W/kg ,

For the upward motion of a column of air over a squaremeter of surface, we have :

- $\dot{P} \approx 0.01 \times 0.7 \times 5000 \times g = 350 \text{ W}$,
- $\dot{E} \approx -0.01 \times 0.7 \times 1000 \times 5000 \times \frac{6.5}{1000} \approx -230 \text{ W}$,
- phase change: $2.2 \times 10^6 \times 10^2 \times \frac{0.04}{24 \times 3600} \approx 100 \text{ W}$,

which is compatible with $W - D = \dot{P} = 350 \text{ W}$ and $\dot{E} = -W + D + Q = -230 \text{ W}$.

The observed lapse rate of 6.5 C/km can be viewed as being obtained by moderating the dry adiabatic rate of 10 C/km by a combined process of phase change and turbulent dissipation effectively reducing the drop of temperature with altitude. The energy transfer in this process ($\approx \frac{3.5}{6.5} \times 230 = 120 \text{ W}$ with $100 - 110 \text{ W}$ for evaporation

and $20 = 0.002 \times 5000 \approx 10 - 20 \text{ W}$ for turbulence) is roughly equal to the heat forcing allocated to thermodynamics ($= 120 \text{ W}$). Increasing heat transfer then corresponds to nonincreasing lapse rate and non warming; the main message of our analysis.

The observed lapse rate of 6.5 C/km is bigger than the moist adiabtic rate of 5 C/km , which causes unstable overturning of rising warm air and turbulent dissipation.

10 Lapse Rate vs Radiative Forcing

If the lapse rate is L then $\dot{P} + \dot{E} = Q$ combined with $\dot{E}/\dot{P} = \frac{L}{10}$ according to the above computation, gives $L = 10(1 - Q/\dot{P})$. If Q is increased then L will decrease if \dot{P} stays constant, but if \dot{P} increases quicker than Q , then L may increase. Increasing Q may be expected to give an increase of \dot{P} by increasing the vertical convection velocity, but a decrease by increasing phase change evaporation/condensation. Which effect will dominate: convection or phase change? Computations with an answer are under way... until then we notice that out of 120 W/m^2 of radiative heat forcing, a major part of say 100 can be allocated to phase change, which gives phase change a good chance to compete with convection...

11 Summary: Atmosphere as Air Conditioner

A good many times I have been present at gatherings of people who, by the standards of the traditional culture, are thought highly educated and who have with considerable gusto been expressing their incredulity at the illiteracy of scientists. Once or twice I have been provoked and have asked the company how many of them could describe the Second Law of Thermodynamics. The response was cold: it was also negative. (C. P. Snow in 1959 Rede Lecture entitled The Two Cultures and the Scientific Revolution).

Let us now sum up the experience from our analysis. We have seen that the atmosphere acts as a thermodynamic air conditioner transporting heat energy from the Earth surface to a TOA under radiative heat forcing. We start from an isentropic stable equilibrium state with lapse rate 9.8 C/km with zero heat forcing and discover the following scenario for the response of the air conditioner under increasing heat forcing:

1. increased heat forcing of the Ocean surface at low latitudes is balanced by increased vaporization,
2. increased vaporization increases the heat capacity which decreases the moist adiabatic lapse rate,
3. if the actual lapse rate is bigger than the actual moist adiabatic rate, then unstable convective overturning is triggered,
4. unstable overturning causes turbulent convection with increased heat transfer.

The atmospheric air conditioner thus may respond to increased heat forcing by (i) increased vaporization decreasing the moist adiabatic lapse rate combined with (ii) increased turbulent convection if the actual lapse rate is bigger than the moist adiabatic lapse rate. This is how a boiling pot of water reacts to increased heating.

If someone points out to you that your pet theory of the universe is in disagreement with Maxwells equations, then so much the worse for Maxwells equations. If it is found to be contradicted by observation, well, these experimentalists do bungle things sometimes. But if your theory is found to be against the second law of thermodynamics 2.2 The Enigma 9 I can give you no hope; there is nothing for it but to collapse in deepest humiliation (Sir Arthur Stanley Eddington in *The Nature of the Physical World*, 1915)

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