STDP between a Pair of Recurrently Connected Neurons with Asynchronous and Synchronous Stimulation

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Abstract

A mammalian nervous system can continuously receive information from the environment. For this, the capability of the neural circuits to dynamically learn and store both short-term and long-term information is essential. Learning and memory occurs through the formation of cell assemblies and the dynamics of their synapses. This thesis focuses on modeling the dynamics of long term memory in a network of two spiking neurons with reciprocal synapses and Spike Timing Dependent Plasticity (STDP).

After preliminary work showed that additive STDP was incapable of forming a recurrent network with a stable structure, the STDP learning rule was modified to be multiplicative and to include a non-Hebbian component. In addition, asynchronous, synchronous, and polysynchronous thalamic stimulation was included in the model. The introduction of different kinds of stimulation, with the contribution of STDP, created the conditions for stable bidirectional synaptic weight growth, stable unidirectional synaptic weight growth and absence of weight growth in either direction. It was found that the firing frequency of the neurons, connection conduction delays and input stimuli are all factors affecting synaptic plasticity and memory storage. Synapses regulated by STDP influenced the timing of the postsynaptic firings. With asynchronous stimulation, STDP contributed to the controlled drift of firing time of the connected neurons. With synchronous stimulation, weight stability was achieved for both feed-forward and recurrent connections, dependent on frequency and connection delays. With polysynchronous stimulation, the neuron stimulated at the lower frequency increases its firing rate to the neuron with the higher frequency. Considering these results, the emergence of strongly connected neurons has a tendency to fire together in a specific temporal window. This may indicate how more complex cell assemblies form in a larger network of neurons.
To my beloved country, IRAN
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1 Introduction

1.1 Short introduction to computational neuroscience

Many different scientific disciplines capture some essentials of brain function such as Machine learning, Neural Networks, Learning Theories etc. Although these research directions try to partially build on brain activity, they do not closely rely on the biological properties of the brain. Neural networks try to represent biology in an abstract sense. While the term ‘Computational Neuroscience’ was not coined by Eric Schwartz until 1985, the initial steps in this field were taken by Andrew Huxley and Alan Lloyd Hodgkin in 1952. Their collaborative work resulted in a novel mathematical model of neural action potentials based on experimental results. Today, computational neuroscience describes and analyses the brain’s capability of processing information. Mathematical and computational modeling of different brain functions are based on the neurobiological aspects and details of neural circuits. Memory and neuroplasticity are fundamental features of brain function and an important direction of research in computational neuroscience.

1.2 From Neurons to Networks

Information received from the environment is transmitted and processed in the brain by cells called neurons. Anatomical investigations of the neurons show that these cells have three main components: soma (cell body), dendrite and axons. The axon and dendrites are cell structures arising from the soma.

Neurons are capable of transmitting information electrically and chemically. Chemical transmission can happen through a synapse (a gap between two connected neurons) via neurotransmitters. The postsynaptic effect is dependent on the released transmitters and on the activated receptors. Chemical synapses can have ionotropic and metabotropic effects. Through ionotropic receptors, the neurotransmitters can either hyperpolarize or depolarize the postsynaptic membrane. If the membrane voltage is depolarized enough to pass a threshold potential, a spike (action potential) occurs. It usually originates from the axon hillock and then the action potential travels along the axon.

Neural connections form a network. A network is composed of many neurons connected in different ways. Some neurons are connected to each other reciprocally, while other neurons just have feed-forward connections. Neurons can also have self-connections, called
autapse, in which case the neuron receives a connection from itself. The time that it takes for a spike to reach the postsynaptic side is called the conduction delay. Any of these three described forms of connection can be found in the brain. In this project, I analyzed the synaptic dynamics of two recurrently connected neurons.

1.3 Outline of the Thesis

This thesis consists of seven chapters. In the first chapter, the motivations of the work are presented. The goals and the assumptions are considered and described. In the second chapter background is provided on different related models and theories. Different spiking models and the theory of synaptic plasticity are explained. Chapter 3 contains the methodology of this research and describes the presented model. The MATLAB simulation results are presented in chapter 4. The discussion is described in chapter 5. Chapter 6 includes the conclusions and possible future directions of the research. The last chapter is the Appendix presenting the MATLAB code simulating the model.

1.4 Background

In the nineteen century, Cajal discovered zones between individual neurons termed ‘synapse’ later by Sherrington. Cajal proposed his novel hypothesis that the information storage in the brain is due the changes of the synaptic efficacy of active neurons, in 1888. Chemical synapses are dynamic, their ‘weights’ can change on different timescales. These changes contribute to the dynamics of the neural network, thus to its ‘adaptability’. Synaptic adaption is the major characteristic of memory storage. Memory is the brain’s ability to encode, store and recall information. This research focuses on memory storage, which can appear both on long and short time scales (long term and short term memory). In contrast to the short term memory, long term memory preserves the encoded information for hours, days or longer.

Donald Hebb presented his novel theory about the correlation between pre- and postsynaptic neurons in 1949, which dramatically influenced existing learning theories. According to the Hebbian learning rule: “When an axon of cell A is near enough to excite a cell B and repeatedly or persistently takes part in firing it, some growth process or metabolic change takes place in one or both cells such that A's efficiency, as one of the cells firing B, is increased.” His opinion about the form and function of cell assemblies can be followed in this statement: "The general idea is an old one, that any two cells or systems of cells that are repeatedly active at the same time will tend to become 'associated', so that activity in one
facilitates activity in the other." The second postulation is often rephrased as “Cells that fire together, wire together”. The degree of correlation between pre- and postsynaptic activity affects the changes of the synapse efficacy\textsuperscript{1,2}. Cell assemblies during learning and ontogenesis emerge as a result of modified connections in the neural networks. Those neurons in a cell assembly, which receive simultaneous stimulation tend to fire together within a certain time window. Neural assembly formation has a key role in memory storage. According to the Hebbian theory, the neural assembly formation is a result of the synaptic modification between simultaneously firing neurons. Synaptic modification can be potentiation or depression. Experimental analysis illustrates the effect of some parameters on synaptic modification.

Over the past years, the roles of firing frequency and spike timing in learning have been debated and investigated\textsuperscript{3,4,5,6}. Initially, many different learning rules have been developed using the mean firing frequency. Hebbian learning focuses on the correlation between pre- and postsynaptic neurons’ firing frequencies. This theory was widely used, for example, the effect of firing rates on receptive field formation was investigated\textsuperscript{7,8}.

Experimental results depict the major role of time in synaptic modification and information processing in the brain. Several experiments (for example from somatosensory system, visual cortex, auditory system and retina) reveal that the spike-timing accuracy has a significance in neural coding and it can be on millisecond, and in some cases on sub-millisecond timescale\textsuperscript{9,10}. Spike-based learning rules are able to capture this precision, in contrast to the rate-based Hebbian learning rules.

Nelson et al\textsuperscript{11} have shown that the sign and the amount of synaptic modification in case of some hippocampal and neocortical synapses depend on firing rate as well as the spike timing of the neurons. Spike Timing Dependent Plasticity (STDP) is a plasticity rule, which takes into consideration the timing of the pre- and postsynaptic neurons firing. This plasticity is a temporal asymmetric form of the Hebbian learning rule. STDP depends on the order and the relative timing of the spiking in pre- and postsynaptic neurons. According to the standard STDP, the order of presynaptic spike arrival and postsynaptic firing and the interval between them influences synaptic weight modification.

This study investigates the contribution of the STDP and non-Hebbian learning rules to the synaptic modification in recurrently connected neurons. Predictions are given for the neuronal assembly formation.
1.5 Motivation

According to the Hebb’s rule the degree of correlation between pre- and postsynaptic neurons determines the synaptic plasticity modification and the correlation was defined in terms of firing rate. Explicitly, he focused on the synaptic weight formation of a feed-forward connection, without analyzing the neural network with feed-forward and recurrent connections. The activity of the cell assembly can be analyzed by studying the activity of its components. Therefore the analysis of synaptic plasticity between two interconnected neurons is relevant both experimentally as well as theoretically and computationally.

As mentioned previously, Hebbian learning rules can be implemented while taking into consideration the firing rate and spike timing. Temporal correlation between pre- and postsynaptic neuron firing affects synaptic weights within milliseconds. This precision and consequently some temporal information is neglected in firing rate learning models. According to the rate-based models, long term potentiation is the result of the simultaneously occurring high firing frequencies in pre- and postsynaptic neurons. In contrast, in case of the STDP rule, presynaptic spike arrival preceding the postsynaptic firing causes synaptic potentiation, regardless of low firing frequencies. It is possible to quantify experimentally the STDP rules and parameters at chemically connected neuron pairs. Thus, the direction and the magnitude of the synaptic plasticity modification is biologically plausible and can be incorporated into the computational models.

Given the above motivations, I studied the STDP rule’s effect on a reciprocally connected pair-wise excitatory neuron case. The STDP model justifies the associative learning intuitively, as the relative timing in STDP introduces ‘causality’. The standard STDP model was designed for feed forward connections and is not capable of forming recurrent connections. The unlimited growth of synaptic weights in classical STDP makes the synaptic connections unstable. This property directed me to modify the classical STDP learning rule.

1.6 Project Evolution

This research project began with the goal of better understanding the process of neural cell assembly formation in order to store input patterns. Part of Eugene M. Izhikevich’s model and implementation was used, which focuses on a network capable of forming polychronous groups. A polychronous group is composed of time locked neurons which do not fire synchronously. In the Izhikevich model, STDP was used as the learning rule. The network was composed of 1000 randomly connected neurons, with a connection probability of 10%
between any two neurons. The randomly generated network consisted of feed-forward, recurrent and self-looping connections. Conduction delays were used in the network and each neuron was assigned a random delay, an integer between 1ms and 20 ms.

In order to activate the neurons, two input sources are implemented in the model. The external source is random thalamic input. During each 1ms time step, a random neuron is chosen and stimulated with a 20 mV amplitude voltage pulse. The other source is the intrinsic synaptic input from other neurons, using a non-Hebbian learning rule. According to neurophysiological observations, the arrival of the presynaptic spike releases neurotransmitters, regardless of the firing of the postsynaptic cell.

The Izhikevich model\textsuperscript{13} applied the standard symmetric STDP model with a temporal window of ±20 ms. However, the standard STDP model has a few shortcomings. The first problem is related to the incapability of additive STDP for achieving bounded and stable synaptic weights. The weights have unlimited growth and do not reach a steady point which is essential for synaptic stability. The second problem is the inability to generate recurrent weights between neurons. The standard STDP model represents the Hebbian learning rule in terms of time, and can be applied to feed-forward connections. Although recurrent connections occurred in the randomly generated network, the applied learning rule could not generate recurrent weight growth in two reciprocal feed-forward connections. This was because the asynchronous stimulation and connection delays only allowed feed-forward propagation of activity. Also, the originally used stimulation paradigm did not represent thalamocortical input and its variations, including synchronous and asynchronous activity. These limitations motivated me to modify the STDP learning rule to produce stable recurrent connections and represent different forms of thalamic stimulation. A network with a large number of neurons is too complicated to track the effects of various parameters on synaptic weight modification. Therefore, the original network of 1000 neurons was reduced to a network of two mutually interconnected neurons.

Clopath et al.\textsuperscript{14} found that a standard STDP rule as used by Song et al.\textsuperscript{15} only produced unidirectional connections, but a modified STDP rule can produce bidirectional connections. For presenting an input pattern as stimulation, the first chosen approach was to use deterministic external voltage pulses with fixed amplitude and intervals, similar to asynchronous input. However, because this was not able to produce both unidirectional and bidirectional connections, synchronous stimulation was also included in the model and simulations.
1.7 Abbreviations and Notation

Table 1.1 presents acronyms used in the thesis and their definitions.

<table>
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<tr>
<th>Acronym</th>
<th>Definition</th>
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<tr>
<td>AMPA</td>
<td>Alpha-Amino-3-Hydroxy-5-Methyl-4-Isoxazole Propionic Acid</td>
</tr>
<tr>
<td>EPSP</td>
<td>Excitatory Post Synaptic Potential</td>
</tr>
<tr>
<td>HH</td>
<td>Hodgkin-Huxley</td>
</tr>
<tr>
<td>ISI</td>
<td>Inter-Stimulus Interval</td>
</tr>
<tr>
<td>LIF</td>
<td>Leaky Integrate and Fire</td>
</tr>
<tr>
<td>LTD</td>
<td>Long Term Depression</td>
</tr>
<tr>
<td>LTP</td>
<td>Long Term Potentiation</td>
</tr>
<tr>
<td>NMDA</td>
<td>N-Methyl-D-Aspartate</td>
</tr>
<tr>
<td>SNN</td>
<td>Spiking Neural Network</td>
</tr>
<tr>
<td>STDP</td>
<td>Spike Timing Dependent Plasticity</td>
</tr>
<tr>
<td>STLR</td>
<td>Spatiotemporal Learning Rule</td>
</tr>
<tr>
<td>VDCCs</td>
<td>voltage dependent calcium channels</td>
</tr>
</tbody>
</table>

Table 1.1. Used Acronym and their definitions

Effective parameters in this thesis have been represented as symbols. Table 1.2 illustrates these parameters and their corresponding mathematical characters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
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<tbody>
<tr>
<td>Number of Neurons</td>
<td>(N)</td>
</tr>
<tr>
<td>Number of Excitatory Neurons</td>
<td>(N_{\text{exc}})</td>
</tr>
<tr>
<td>Membrane Time Constant</td>
<td>(\tau_m)</td>
</tr>
<tr>
<td>Potentiation time constant</td>
<td>(\tau_+)</td>
</tr>
<tr>
<td>Depression time constant</td>
<td>(\tau_-)</td>
</tr>
<tr>
<td>Synaptic Time Constant</td>
<td>(\tau_s)</td>
</tr>
<tr>
<td>Spiking Threshold</td>
<td>(V_{\text{th}})</td>
</tr>
<tr>
<td>Resting Membrane Potential</td>
<td>(V_r)</td>
</tr>
<tr>
<td>Synaptic Weight</td>
<td>(w_t)</td>
</tr>
<tr>
<td>Thalamocortical Input</td>
<td>(I_{\text{th}})</td>
</tr>
<tr>
<td>Intrinsic Synaptic Input</td>
<td>(I_{\text{in}})</td>
</tr>
<tr>
<td>Maximum potentiation amplitude</td>
<td>(\Lambda_+)</td>
</tr>
<tr>
<td>Maximum depression amplitude</td>
<td>(\Lambda_-)</td>
</tr>
<tr>
<td>Firing Rate</td>
<td>(r)</td>
</tr>
</tbody>
</table>

Table 1.2. Common parameters and their symbols
Neurons, Networks and Plasticity

2.1 Spiking Neural Networks

Neural network models and simulations aim to capture certain aspects of the nervous systems’ function. A more recently developed type of neural network is Spiking Neural Network (SNN). In contrast with the previous generations of neural networks, SNN considers the dynamics of neurons based on more detailed biological facts. Non-spiking models can symbolize the neural activity with discrete output [0 1] while spiking models utilize voltage, current, conductance etc. variables to describe the neuronal dynamics. The roles of time and firing frequency become important in information encoding.

2.1.1 Integrate and Fire

One of the first neural models was introduced by Louis Lapicque. He linearly modeled the subthreshold neuronal dynamics. He used a simple circuit model of resistance and capacitance. The name describes the model’s dynamics. When the membrane potential reaches a threshold value, the neuron fires. After the spike, the voltage is reset to a new membrane potential value. A higher current level leads to a higher firing frequency. In this model, there is no upper bound for firing frequency. This limitation can be overcome by adding a refractory period to the model, preventing the firing frequency from reaching arbitrarily high values. A further limitation of this model is that the memory of the neuron is time independent. To solve this problem, a ‘leak’ term was introduced to the membrane potential, reflecting the diffusion of ions that occurs through the membrane, when the system is not in equilibrium. The following equation describes the neuron’s inter-spike dynamics:\(^{42}\):

\[
C_m \frac{dV_m}{dt} + \frac{V(t)}{R} = I(t)
\]  

(2.1)

Although, the ‘integrate and fire’ model computationally is efficient, biologically it is too simple and cannot describe the cell dynamics.

2.1.2 Hodgkin-Huxley Model

In 1952, a neural model was presented by Alan Lloyd Hodgkin and Andrew Huxley, called the Hodgkin-Huxley (HH) model. They proposed a model which captures the process of neural excitability and action potential generation. The model is based on the time and voltage dependent ion channel dynamics, and was able to simulate the eletrophysiological recordings measured in the giant squid axon. In the HH model electrical properties are
approximated by coupled nonlinear differential equations. According to the HH model, a neuron resembles an electrical circuit. The electrical activity of a neuron is modeled as follows:

\[ C_m \frac{dV_m}{dt} + I_{ion} = I_{ext} \]  
\hspace{1cm} (2.2)

Where, \( C_m \) is the membrane capacitance, \( V_m \) is membrane potential, \( I_{ion} \) is the ionic current and \( I_{ext} \) is the injected external current. The ionic current, \( I_{ion} \), in the HH model has three components, sodium, potassium and leak current.

\[ I_{ion} = G_{Na}(V_m - E_{Na}) + G_{k}(V_m - E_{k}) + G_{L}(V_m - E_{L}) \]  
\hspace{1cm} (2.3)

Where \( G \) in the above equation represents the conductance of each ion and \( E \) is the equilibrium potential. They defined two states for the gating variables: permissive or non-permissive. Channels can be open or closed and their state depends on the membrane potential and time. A channel is open and lets the ions flow through it, if all the gates are in the permissive state. If even one of the gates are in the non-permissive state, the channel is closed. The ion channel conductance can be calculated according to the following equations:

\[ G_{Na} = \tilde{g}_{Na} p_m^3 p_h \equiv \tilde{g}_{Na} m^3 h \]  
\hspace{1cm} (2.4)

\[ G_{k} = \tilde{g}_{k} p_h^4 \equiv \tilde{g}_{Na} n^4 \]  
\hspace{1cm} (2.5)

In the model, two types of gating variables are used for the \( Na^+ \) ion: \( m \) and \( h \). According to their experimental findings, Hodgkin and Huxley estimated three \( m \) gates for activation and one \( h \) gate for inactivation of the \( Na^+ \) channel. Similarly, the \( K^+ \) channel is composed of four \( n \) gates for activation. \( \alpha \) and \( \beta \) are the voltage dependent rate constant representing the transition of non-permissive to permissive states and vice versa. The transition probability follows the non-linear differential equations described in the following formulas:\(^{43}\).

\[ \frac{dm}{dt} = \alpha_m(V)(1 - m) - \beta_m(V)m \]  
\hspace{1cm} (2.6)

\[ \frac{dh}{dt} = \alpha_h(V)(1 - h) - \beta_h(V)h \]  
\hspace{1cm} (2.7)

\[ \frac{dn}{dt} = \alpha_n(V)(1 - n) - \beta_n(V)n \]  
\hspace{1cm} (2.8)

### 2.1.3 Izhikevich model

Of the two spiking models were introduced previously, the LIF is computationally efficient but biologically less plausible while the HH model more accurately describes neural excitability but is computationally expensive. In 1961-1962, the FitzHugh-Hagumo model simplified the HH model as a two-dimensional system:\(^{40}\).
Izhikevich proposed a model, where he combined certain properties of the above models. His model is able to produce both spiking activity and bursting. Different firing patterns are presented in Figure 2.1. The following equations were used in his model:

\[ v' = v + 0.04v^2 + 5v + 140 + u + I \]  \hspace{1cm} (2.9)

\[ u' = a(bv - u) \]  \hspace{1cm} (2.10)

Where \( v \) represents the membrane potential of the neuron \( u \) is a variable which describes the membrane recovery, which corresponds to the \( k^+ \) activation and \( Na^+ \) inactivation. Equation (2.11) describes the variable changes when the \( v \) (membrane potential) reaches a threshold value.

\[ \text{if } v \geq +30 \text{ mV, then } \begin{cases} v \leftarrow c \\ u \leftarrow u + d \end{cases} \]  \hspace{1cm} (2.11)

The firing patterns depend on the parameters \( a, b, c \) and \( d \). The \( a \) and \( b \) parameters influence the voltage growing before reaching a threshold. The \( c \) and \( d \) parameters are responsible for the neuron behavior after emitting an action potential. Depending on the neuron type, (for example excitatory or inhibitory), each parameter \((a, d)\) receives a special value.

The membrane potential is reset once it reaches the spike detection value of +30 mV. It is important to mention that this value corresponds to the voltage of the action potential peak controlling the spiking dynamics. Similarly, as in the case of LIF, the action potential initiated as soon as the membrane potential reaches the threshold. But in contrast with the LIF, the threshold is not fixed but can vary between -40 and -55 depending on the membrane potential history before the most recent spike.
2.2 Synaptic plasticity

2.2.1 Context of synaptic plasticity

Synaptic plasticity is the fundamental feature of most neural networks. A cortical map, learning and memory storage, sensory processing are phenomena, strongly influenced by various forms of synaptic plasticity. Malenka\textsuperscript{15} categorized the plasticity mechanisms into three groups: (a) modification of synaptic efficacies (b) generation of a new connection or elimination of exciting synapses (c) excitability adaption of individual neuron. My thesis investigates some computational consequences of the synaptic plasticity changes, which belong to the first category. Synaptic modification can be categorized according to three criteria: Direction, Duration and Source of induction. Synaptic modification can have two directions: growth (potentiation) and decrease (depression). Growth of synaptic strength of an excitatory synapse results in a depolarization of the postsynaptic membrane potential, making it more probable that postsynaptic neuron will fire. In contrast, synaptic depression of an excitatory synapse corresponds to decreased depolarization of the postsynaptic membrane potential. The synaptic modification can be divided into two categories based on time: long term and short term synaptic plasticity. In the case of short term synaptic plasticity, synaptic changes last for seconds and the changes are dependent on the presynaptic spike trains, but not on the postsynaptic state. Long term synaptic plasticity lasts from minutes to hours and more, and this change is influenced by both presynaptic spike trains, and the postsynaptic state.

![Figure 2.2. Illustration of different parameters on long term modification\textsuperscript{39}.](image)
According to the direction of plasticity two categories are defined: long term potentiation and long term depression. Experimental observations described different parameters of presynaptic and postsynaptic cells in different parts of the brain\textsuperscript{16}. These differences lead to various forms of synaptic plasticity. Figure 2.2 is a simple schematic of long term modification. Presynaptic firing rate, postsynaptic membrane potential and calcium concentration are represented as influential factors. Besides the above factors, ‘spike timing’ is also a significant and influential factor in synaptic modification. The following chapter section discusses the effect of spike timing on synaptic plasticity.

### 2.2.2 Biological Mechanism of Neuroplasticity

The biological mechanisms of long term modification typically involve the glutamate receptors NMDA and AMPA. NMDA is a non-specific cation receptor, permeable to Na\(^+\), K\(^+\) and Ca\(^{+2}\) and can be seen on both presynaptic and postsynaptic cells but AMPA receptors, permeable to Na\(^+\) are embedded only in postsynaptic membranes. The released presynaptic glutamate binds to AMPA receptors and opens them. The NMDA receptor has a Mg\(^{+2}\) ion in it at resting potential. When the membrane depolarizes, the back-propagated action potential make Mg\(^{2+}\) be separated from the NMDA receptor and opens the receptors. As glutamate binds to the AMPA receptor, the AMPA receptor opens increases the postsynaptic membrane potential. This causes a detachment of Mg\(^{+2}\) from the NMDA receptor. The open NMDA receptors allow the cation influx. The postsynaptic Ca\(^{+2}\) concentration determines and influences the long term synaptic modification.

Homosynaptic interaction represents the synaptic modification in one synapse, (junction of two neurons). Heterosynaptic interaction and modification is present at the connections and interactions in between different synapses.

### 2.2.3 Phenomenological models

Thewlis in the Concise Dictionary of Physics writes a definition for phenomenological theory:

“A theory that expresses mathematically the results of observed phenomena without paying detailed attention to their fundamental significance.”

Phenomenological models of synaptic plasticity assume synaptic plasticity as a black box such that some inputs are introduced to and synaptic efficacy modification are generated as its output. Although biophysical models analyzing the neural activity are accurate, computationally they are not efficient. Phenomenological models just focus on the
mathematical aspect of the behavior of the neuron, and they are computationally tractable. Figure 2.3 depicts the decay and update states of a hidden variable.

![Graph](image)

*Figure 2.3 The contribution of spikes on the trace of hidden variable, x. The top figure illustrates the growth of each spike with a fixed value and the bottom shows its update to a fixed value.*

The synaptic modification is influenced by the correlation between the pre- and postsynaptic cells, but do not have immediate influence on the synaptic weight. This delay can be described by the update of a hidden variable, x. The spike of the x at time \( t_i \) can be presented by delta function \( \delta(t - t_i) \). The updated variable with the magnitude A decays with time constant of \( \tau \). The update of spike can be controlled by applying saturation. The variable x is bounded between the [0 1] for A<1.

\[
\frac{dx}{dt} = -\frac{x}{\tau} + \sum_{t'} A \delta(t - t') \quad (2.12)
\]

\[
\frac{dx}{dt} = -\frac{x}{\tau} + \sum_{t'} A (1 - x_{-}) \delta(t - t') \quad (2.13)
\]

Experimental results demonstrate the effect of various parameters in synaptic plasticity such as presynaptic firing rate, postsynaptic membrane potential, spikes interval and calcium flow\(^9\). As it was described before, in most classical studies, the correlation between pre- and postsynaptic neuronal activity are based on the correlation between firing frequencies of these neurons. Rate-based models pass over pre/postsynaptic spikes correlation in time with fine resolution. The Hebbian learning rule has a spike-based version. Time plays a crucial role in spike-based learning rules. The model can be defined and formulated according to the input type. Therefore, two different model families were developed previously: rate-based and spike-based model. Before expanding on these two model types, Hebbian theory should be presented.

### 2.2.4 Hebbian Learning Rule

Donald Hebb was a Canadian psychologist who presented a novel idea about the process of learning and memory. Regarding his theory, when a presynaptic cell i repeatedly stimulates its postsynaptic cell, j, the neuron i’s efficiency increases. The simultaneous activity of two neurons increases the tendency of the neurons to become associated and form a ‘Cell
Assembly'. This statement can be rephrased as well: “Neurons that fire together wire together”. Three concepts are implicitly mentioned in this theory: cooperativity, causality and locality. According to Abbott and Gerstner\textsuperscript{17}, a learning rule must be able to satisfy at least six requirements: Localcy, Cooperativity, Synaptic depression, Boundness, Competition and long term stability. According to the parameters that affect the synaptic modification, Hebbian learning rules can be categorized in two classes: rate-based and spike-based.

2.2.4.1 Rate-Based Learning Rules

In order to clarify Hebb’s postulation mathematically, special attention is paid to synaptic efficacy, $w_{ij}$, which can be correlated to the amount of transmitters passing through the synapse from neuron $i$ to neuron $j$. The output of each neuron is represented by their firing rate, $v_i$ and $v_j$. Abbott and Gerstner\textsuperscript{17} describe this aspect of Hebbian learning in detail as follows:

It is important to consider that the applied model analyses the synaptic weight change of a specific presynaptic cell among many cells that all end on one postsynaptic cell. Therefore, mathematically the behavior of the postsynaptic cell can be modeled very simply as follows\textsuperscript{17}:

\begin{equation}
    v_{j}^{post} = \sum_{n=i}^{k} w_{nj} v_{n}^{pre}
\end{equation}

\begin{equation}
    \frac{dw_{ij}}{dt} = F (w_{ij}; v_i, v_j)
\end{equation}

Where $F$ is not known yet. The first aspect mentioned above which should be considered is locality, which corresponds to the homosynaptic aspect of a synapse. Synaptic modification, $w_{ij}$, depends on the activity of its presynaptic cell, $i$ and postsynaptic cell, $j$ and cannot be affected by other neurons, e.g. $k$, even if one ends on same postsynaptic cell. In this model, synaptic modification is a function of firing rates and $\frac{dw_{ij}}{dt}$ is the rate of synaptic weight change. An interesting point in the above function is that postsynaptic membrane potential is not included as an independent variable. Membrane potential, $v_j$ can be calculated as a function of the postsynaptic firing rate.

\begin{equation}
    v_j = g(u_j)
\end{equation}

As mentioned above, there are some properties of synaptic plasticity that any model should capture. Therefore, function $F$ can be presented\textsuperscript{17} by Taylor series around $v_i = v_j = 0$. 

13
\[
\frac{dw_{ij}}{dt} = c_0(w_{ij}) + c_1^{\text{post}}(w_{ij})v_i + c_1^{\text{pre}}(w_{ij})v_j + c_2^{\text{pre}}(w_{ij})v_i^2 + c_2^{\text{post}}(w_{ij})v_j^2 + c_2^{\text{corr}}(w_{ij})v_i v_j + O(v^3) \tag{2.17}
\]

Cooperation between two neurons shows that both neurons should be active to be able to change the synaptic weight. \( c_2^{\text{corr}}(w_{ij})v_i v_j \) with \( c_2^{\text{corr}} > 0 \) is the term which describes the role of cooperativity between two neurons. The first sum in equation 2.17 gets its value in order to satisfy the depression feature of a learning rule. The second and third first order terms are non-Hebbian and correspond to the effect of pre or postsynaptic firing, regardless of post or presynaptic firing respectively. These two terms are explained in detail mathematically in part ‘non-Hebbian learning rules’.

The synaptic weight should be bounded by an upper and lower bounds: \( 0 \leq w_{ij} \leq w_{\text{max}} \) where \( w_{\text{max}} \) is the maximum strength of synapse. As mentioned before, change of the synaptic weight is a function of weight. This can limit the weight growth. The independency can lead to unlimited growth and consequently synaptic saturation. In order control the weight change, a coefficient \( c_2^{\text{corr}} \) can be defined as follows\(^\text{17}\):

\[
c_2^{\text{corr}}(w_{ij}) = \gamma_2 (w_{\text{max}} - w_{ij})
\]

(2.18)

Growth of the weight with \( \gamma_2 > 0 \) results in coefficient reduction and decrease of the weight makes the coefficient larger. This form of constraint is called ‘soft boundary’ Chapter 3 describes different kinds of boundaries in detail. Hebb focused on the positive synaptic modification and his idea does not cover the synaptic depression. Obviously, depression is an essential phenomenon in synaptic modification. In order to capture synaptic depression, a negative term should be added to the formula\(^\text{17}\).

\[
c_0(w_{ij}) = -\gamma_0 w_{ij}
\]

(2.19)

Competition is another aspect which should be discussed. Competition among the presynaptic neurons which are converging on the same postsynaptic neuron, result in growth of some weights (Homosynaptic LTP) at the expense of other synaptic weights decline (Heterosynaptic LTD). Abbott and Gerstner\(^\text{17}\) mentioned that a learning rule should cover this process without any extra assumptions. Competition among the synapses which converge on the same postsynaptic cell can be implemented by weight normalization. There are different forms of weight normalizations. Normalizing on a weight sum, \( \sum_t w_{ij} \) should be avoided since another parameter should be introduced. Another form of normalization which is applicable here is normalizing on a quadratic form of weights. The quadratic norm of weight vector \( |w_{ij}^2| \), is equal to the length of the weight vector and is constant. Therefore, by applying this normalization, an increase of a synaptic strength causes a decrease of other
synaptic weights. The weight vector has $N$ dimensions; each representing a synaptic weight on the common postsynaptic cell. In 1982, Finish computer scientist Erkki Oja, modified the classical Hebbian algorithm, improving his unrestricted learning rule to include synaptic saturation. In order to stabilize the weight, he proposed\textsuperscript{17} a subtraction term to subtract some value proportional to the second order of the postsynaptic output, $v_j^2$.

$$\frac{dw_{ij}}{dt} = \gamma_2 \left( w_{\text{max}} - w_{ij} \right) v_i v_j - \gamma w_{ij} v_j^2$$ \hspace{1cm} (2.20)

The last concept that should be mentioned in designing a learning algorithm is long term stability. It is clear that when the training phase of the network is over, the trained synaptic strengths are fixed. But in a dynamical network, inputs and states can change. Therefore, it is vital for the network to keep its previously stored information and not get overwritten. In order to consolidate the previously learned weights, normalized weights should be introduced. This idea can be implemented in the following equation\textsuperscript{17}:

$$c_0(w_{ij}) = -\gamma_0 w_{ij} \left( 1 - w_{ij} \right) \left( w_\theta - w_{ij} \right)$$ \hspace{1cm} (2.21)

Where $0 < w_\theta < 1$ and $\gamma_0 > 0$. In the above sections, some aspects and limitations were covered, which emerge in Hebbian theory. In order to improve Hebbian learning rule, two rate-based models emerged from the BCM model, discussed next

**BCM rule**

In 1981, Bienenstock, Cooper and Munro (BCM) proposed a new model\textsuperscript{17} to compensate the Hebbian model deficiencies. In their opinion, the synaptic modification is the result of postsynaptic cell activity and not the function of the presynaptic neuron. According to their results, if the postsynaptic cell firing rate is more than the threshold $v_\theta$ the synapse is potentiated long-term and if it is less than the threshold, long-term synaptic depression occurs. This model cannot control the perpetual growth of synaptic strength. Therefore, they introduced the notion 'sliding threshold' which is a function of the history of postsynaptic activity. The BCM model is presented in the following equation\textsuperscript{17}:

$$\frac{dw_{ij}}{dt} = \eta \phi(v_j - v_\theta) v_i - \gamma w_{ij}$$ \hspace{1cm} (2.22)

BCM theory was developed to model the direction tuning and ocular dominance in cat and monkeys. Synaptic modification in many different experimental conditions can be described by a BCM type learning model, for example what has been found in hippocampus and visual cortex.
2.2.4.2 Spike-Based Learning Rule

As mentioned in the previous section, synaptic modification is a function of presynaptic and postsynaptic cells behavior and the strength of connections between them. Firing rate is the only considered output of a neuron in rate-based rules. Therefore, synaptic modification has been calculated based on the pre- and postsynaptic cell firing rate. In contrast, spike-based rules analyze the synaptic modification on the level of the individual spikes. The temporal order of pre- and postsynaptic activity and the number of spikes in a given time window are the main considered parameters. The neural spike train is modeled as \( S_i(t) = \sum_j \delta(t - t^f) \). The variable \( t^f \) is the time that neuron \( i \) fired. Equation (2.15 can be reformulated based on individual spikes as follows\(^{17} \):

\[
\frac{dw_{ij}}{dt} = F(w_{ij}, S_i(t'), S_j(t''))
\]

\( t' \) and \( t'' \) refer to the firing frequency of pre- and postsynaptic cell, respectively. Similarly as in the previous section, this learning rule can be expanded by Taylor series\(^{17} \).

\[\Delta w_{ij} = c_0(w_{ij})T + \sum_{n} c_{i,j}^{pr} + \sum_{n} c_{i,j}^{post} + \sum_{n} c_{i,j} W(t_{i}^{(f)} - t_{j}^{(f)}) + \ldots \] (2.24)

The first sum on right hand side of equation 2.24 represents the activity independent synaptic plasticity. This coefficient is dependent to the synaptic weight but independent of neurons firing. The next two terms represent the effect of the pre- and postsynaptic neurons on the synapses regardless of the effect of their partner (post and presynaptic cell, respectively). The last term describes the correlation between the two cells’ activity. Spike Timing Dependent Plasticity emerged from the spike-based Hebbian learning rule. In following section, this issue is described in detail.

2.2.5 Spiking Timing Dependent Plasticity

2.2.5.1 Classical Overview

Nelson et al.\(^{11} \) mentioned that at hippocampal and neocortical synapses the sign and the amount of synaptic modification (plasticity) depend on both the firing rate and spike timing. The significance of spike timing, created a new concept in the plasticity field which led to the development of STDP learning rules.

W. Levy and O. Steward were the pioneers in the field of associative memory in 1983\(^{18} \). They did some experiments to analyze the role of time in synaptic plasticity. From then on, many others examined the relationship between the pre- and postsynaptic spikes in terms of time. Debanne et al.\(^{22} \) in 1994 observed the depression of the synaptic weight due to the
asynchronous activity of pre- and postsynaptic cells. Henry Markram found experimental evidence for the STDP model in 1997\textsuperscript{19}. He repetitively stimulated presynaptic neuron 10 ms before the postsynaptic neuron’s stimulation. Synaptic growth was the result of this order of firing with the mentioned interval. When he changed the order of stimulation with the same interval, synaptic depression emerged. His experiments were performed using a dual patch clamping technique.

The described phenomenon is a temporal asymmetric form of Hebbian learning rule. STDP follows two different forms of long term plasticity, dependent on the order of spike timing. The order of presynaptic spike arrival and postsynaptic firing can determine the direction of modification. If the presynaptic spike arrives at the synaptic terminal a few milliseconds before postsynaptic firing, the synapse is potentiated. If the order is in the opposite direction and postsynaptic firing occurs a few milliseconds after the presynaptic spike arrival the synapse is depressed. The magnitude of modification is the result of the temporal distance between the activity of pre- and postsynaptic cells. The classic form of STDP can be described as follows\textsuperscript{16}:

\[
W(\Delta t) = F(\Delta t) = \begin{cases} 
A_+ \exp(\Delta t / \tau_+) & \text{if } \Delta t < 0 \\
-A_- \exp(\Delta t / \tau_-) & \text{if } \Delta t \geq 0 
\end{cases} \tag{2.25}
\]

\(A_+\) and \(A_-\) are the maximum amount of synaptic modification occurring when \(\Delta t\) comes to zero. \(\tau_+\) and \(\tau_-\) are time constants symbolizing the decay of STDP window\textsuperscript{12}. There are different opinions about the effective time window of STDP. Some researchers emphasize an equal depression and potentiation time domain,\textsuperscript{\textsuperscript{\textsuperscript{\textsuperscript{\textsuperscript{\textsuperscript{20}}}}}20\text{ ms}}, but others believe that the depression time window should be larger than potentiation, resulting in a generally stronger depression than potentiation, LTP time window \(\leq 20\text{ ms}\) and \(-50 \leq \text{ LTD time window} < 0\) and there is no modification outside of these intervals. The balance or imbalance between potentiation and depression has been always a challenging topic in STDP which is described in the following sections. Most experimental data reveal that the potentiation time window is almost equal to the depression time window\textsuperscript{19, 20, 21}. \textsuperscript{\textsuperscript{\textsuperscript{\textsuperscript{\textsuperscript{\textsuperscript{21}}}}}}
Figure 2.4: Spike Timing Dependent Plasticity modification function. Synaptic change due to pair-based activity of pre/post synaptic side.

Nevertheless, some other experiments declare that the temporal window of the negative part of STDP is longer than the positive one and depression overcomes potentiation. Figure 2.4 shows the depression and potentiation in the imbalance form of STDP. During the time, classical STDP is modified to improve its limitations. Different models are present in the following sections.

2.2.5.2 Biological Aspects of STDP

According to the classical point of view, long term depression is the result of low firing frequency of a presynaptic cell and high firing frequency make the synapse potentiated while the postsynaptic firing rate is constant. STDP opened new ways for describing LTP and LTD. According to this new concept, low firing frequency may cause depression or potentiation. The order of spikes sequence results in LTP or LTD.

As described before and as Figure 2.5 shows, the role of NMDA receptors and the concentration level of Ca^{2+} influx are the same, but the role of postsynaptic spike in calcium concentration change is different. According to their experimental observations, Koster et al. presented two definitions related to synaptic modification: supralinear and sublinear summation of calcium influx. They concluded that the order of EPSP and postsynaptic AP both influence the level of calcium influx. As is illustrated in Figure 2.5 each of EPSP and postsynaptic AP has a specific effect on the calcium concentration. Experiments demonstrate that the order of pre/post spikes result in supralinear summation of Ca^{2+} influx through the NMDA receptors and voltage dependent calcium channels (VDCCs) sublinear summation of Ca^{2+} influx was observed as a consequence of post/presynaptic order.
Figure 2.5. Synaptic transmission. Left: during resting potential. Right: During postsynaptic depolarization.

The NMDA receptor is known as a molecular coincidence detector. Neural coincidence detection is a process in which a neuron or a group of neurons can encode the transformed information from two separate inputs, based on the nearly simultaneous arrival time. The opening of NMDA channels are dependent on the separation of Mg$^{2+}$ and is the result of glutamate binding and postsynaptic depolarization. Experimental results show that the magnesium detachment is the process that can happen fast or slow. The rate of Mg$^{2+}$ removal depends on the order of glutamate binding and on the cell depolarization. The long term changes of synaptic weight are due to the activation of NMDA receptors. When the postsynaptic membrane potential depolarizes just a few milliseconds after the binding of glutamate to the NMDA receptor, magnesium releases from the receptor so fast. The relief of Mg$^{2+}$ happens due to the back-propagation of the action potential. The detachment of Mg$^{2+}$, increases the Ca$^{2+}$ influx through NMDA receptors and consequently strengthen the synaptic weight.

Some aspects of STDP can justify the long term depression according to the order of post-preshypaptic firing. First, presynaptic spike arrival after the postsynaptic action potential causes afterdepolarization in membrane potential, resulting in the partial opening of NMDA receptors and consequently low level of Ca$^{2+}$ influx. Therefore, synaptic depression occurs. The other is finding another coincidence detector independent to NMDA receptor. Karmarkar and Buonomano in their experiments found amGluR pathway and they describe VDCCs dependent Ca$^{2+}$ flux. They propose this mechanism as another probable coincident detector.
2.2.5.3 Nearest Neighbor versus All-to-ALL

Classical STDP analyses the synaptic changes according to the present pre/ post synaptic spikes and their inter-spike interval (ISI). STDP can be assessed in two different groups: History independent and History dependent. The former which is also called, ‘nearest neighbor’, refers to the effect of the latest pre/post spikes on synaptic strength. The classical form of STDP focuses on individual pair of spikes, described in equation (2.25). The other category is talking about the effect of previous spikes of the latest one, and how do they effect the synaptic weight. Here, the individual spike is not effective for influencing the synaptic weights, but a burst of spikes are considered. Two approaches are presented for describing the effect of firing history on the synapses: Suppression model and revised suppression model. The former model considers only one spike before the latest pre/post synaptic spikes. In this case, the efficacy of each spike is affected by its previous spike. This is called original suppression model. The efficacies of penultimate spikes should be calculated:

\[\varepsilon_k = 1 - e^{-\frac{(t_k - t_{k-1})}{\tau_s}}\]  \hspace{1cm} (2.26)

\[W(\Delta t) = \varepsilon_{pre} \varepsilon_{post} F(\Delta t)\]  \hspace{1cm} (2.27)

where \(t_k, t_{k-1}\) are the \(k^{th}\) and \((k-1)^{th}\) spike of the neuron, respectively. The calculated efficacy, \(\varepsilon_k\), is the same for pre- and postsynaptic cells.
In STDP literature, the later the form is also called ‘all to all’ as well. In this model the firing pattern of each neuron should be considered. The firing history of each neuron affects the efficacy ($e$) of the subsequent spikes. This model is called Revised Suppression model. The efficacy of each presynaptic spike not only affected by its immediately previous spike but also by all the previous spikes. Therefore,

$$e_{pre} = \prod_{j=1}^{i-1} \left( 1 - e^{-\frac{(t_i-t_{i,j})}{\tau_e}} \right)$$

The value of postsynaptic efficacy should be derived according to the original suppression model. Figure 2.7 shows two different forms of STDP rules in terms of influential spikes on synaptic modification. The figure on top shows the ‘nearest neighbor’ and figures on bottom show the ‘all to all’ form of STDP.

### 2.2.5.4 Additive versus Multiplicative STDP

One problem with the original Hebbian learning rule is the synaptic unlimited growth, as it was described in the previous section. Thus, some bound should be applied for synaptic strength to prevent its saturation. This idea can be implemented by different approaches to calculate the synaptic modification in STDP model: Additive and multiplicative. The difference between these two approaches is in the dependency of the synaptic change. The first approach calculates the synaptic modification, independent to its present weight. According to the classical STDP rule, equation (2.25), $A_+$ and $A_-$ show the maximum synaptic modification. These two positive variables can be independent and constant as presented in by Song et al. Dominance of synaptic depression over potentiation resulted in $A_-/A_+ = 1.05$ and in order to simulate, they set $A_+ = 0.005$. Figure 2.4 represents synaptic weight changing based on additive STDP rules.

The second approach is formulated such that the synaptic strength update is dependent on the present weight value. This approach uses both hard bound and soft bound constraints for the growth and decline of synaptic strength. Soft bounds constraints the rate of synaptic change to the maximum and minimum values. This constraint weakens the increment (decrement) of strong (weak) synapses to their maximum (minimum) values. This boundary is implemented with variables $A_+$ and $A_-$ as written below:
Another constraint that controls the synaptic weight growth is called ‘hard bound’. The synaptic weight is limited by the maximum and minimum values, \( 0 \leq w_{ij} \leq w_{\text{max}} \).

### 2.2.5.5 Shifted STDP

Babadi \(^{36}\) states that neural stability and competition are incompatible with each other. He presented a new STDP model to solve this problem. He proposed that shifted STDP solve this problem easily. In this case there is no need to use the multiplicative STDP. The equation below describes the shifted STDP\(^ {36}\):

\[
W(\Delta t) = F(\Delta t) = \begin{cases} 
A_+ \exp\left(\frac{\Delta t - d}{\tau_+}\right) & \text{if } \Delta t < d \\
-A_- \exp\left(\frac{\Delta t - d}{\tau_-}\right) & \text{if } \Delta t \geq d 
\end{cases}
\]  

(2.32)

\(d\) is the positive parameter that represents the shift of window. The different models of STDP described above can be seen schematically in Figure 2.8.

**Figure 2.8 Different STDP models.**

### 2.2.6 Non-Hebbian Learning Rule

In 1996, Tsukada et al. introduced a theory that was in contrast with the Hebbian learning rule. According to Hebbian theory, the synaptic modification is due to the correlation between the pre- and postsynaptic neurons firing. In contrast, the non-Hebbian learning rule focuses on the synaptic modification due to the presynaptic effect regardless of postsynaptic firing. The spatiotemporal learning rule (STLR) is one form of a non-Hebbian learning rule. Based on their experimental findings, Tsukada et al. \(^{29}\) claim that both STLR and Hebbian learning rules can be found in Hippocampal CA1 pyramidal cells.
Cooperative plasticity is another term coined with this model. Recent studies proved that postsynaptic membrane potential changes are not due to the involvement of one synapse, but due to the cooperation and association of different synapses that are spatially close to each other.

Figure 2.9. Associative long term potentiation, red projects the strong inputs and pink presents the weak one: (a) two groups of presynaptic neurons convergent to one group of postsynaptic cells. (b) Strong synapses. (c) Weak synapses. (d) Stimulation of postsynaptic cell through the weak and strong presynaptic cells. (e) Effect of cooperation of weak and strong cell, associative LTP.

Figure 2.9 illustrates the associative LTP. Each circle represents a group of neurons. The red circle shows strong presynaptic inputs, which results in generation of strong synapses. Therefore, the postsynaptic side is stimulated strongly. The pink circle symbolizes the weak presynaptic inputs. Figure 2.9 (b) and (c) show the individual effect of strong and weak presynaptic inputs, while figure (d) presents the cooperation between these two groups. This association results in strengthening the connection between the weak presynaptic groups and postsynaptic side. It is important to mention that the temporal aspect of cooperation should be considered. According to the described concept, STLR covers two factors: “associative plasticity among presynaptic cell without the involvement of postsynaptic spikes” and “their temporal summation.” These two concepts are reflected in the following equations:

\[
I_{ij}(t) = w_{ij}(t)x_i(t)\sum_{k \neq i}(w_{kj}(t)x_k(t)) \tag{2.33}
\]

\[
\Delta W_{ij}(t) = \eta h \left( \sum_{m=0}^{n} I_{ij}(t_m)e^{-\lambda_2(t-t_{n-m})} - \theta \right) \tag{2.34}
\]

Equation (2.33) defines the value of cooperative activity among the inputs. Figure 2.10 illustrates two underlying factors of STLR schematically.
$W_{ij}$ is the synaptic weight between the presynaptic neuron $i$ and postsynaptic side, $j$. 
$\Delta W_{ij}(t) = W_{ij}(t + 1) - W_{ij}(t)$. Where $\eta$ symbolizes the learning rate coefficient. $x_i(t)$ is the degree of excitation of presynaptic inputs to neuron $j$. $y_j(t)$ represents the output. $h(u)$ is the sigmoid function and $h$ is the threshold of this function. Finally, $k_2$ is an exponential decay time constant.

2.2.7 From Thalamus to Cerebral Cortex

In this thesis, mostly sensory cortical neurons are considered. Cortical neurons are connected to each other in a feed-forward or recurrent manner. In proportion to the cortical area that these neurons belong to, they are stimulated by feed-forward thalamic inputs. Except for the olfactory system, all the sensory systems send their sensory signals to the associated thalamic nucleus and the stimulated nucleus relays the signal back to the related cortical area. The described path is shown in Figure 2.11. An interesting issue that attracts significant attention is spatiotemporal dynamics of the thalamocortical oscillations. The degree of synchrony is assumed to be an effective parameter on forming cortical patterns.

![](image)

**Figure 2.11.** Thalamocortical (TC) cells receive prethalamic (Pre) afferent connections which can be the sensory afferent and relayed to the associated cortical area. The thalamocortical connection arborizes in layers I, II, IV and VI. The corticothalamic connection rooted from Pyramidal (PY) cells and project to thalamic cells.

Wang and et al.\textsuperscript{31} and Minlebaev and et al.\textsuperscript{32} focused on the importance of synchrony among the thalamocortical oscillations to increase cortical reliability and Destexhe and Seinowski\textsuperscript{33} emphasized the active role of cortex on the synchronous thalamocortical input.
3 Model Description

3.1 Methodology

In order to form a memory in a network, neurons can be trained with an activation pattern. The memory storage depends on neural synaptic growth and the generation of strong connections. In networks with many connections, it is difficult to understand the influence of connections on each other, and this makes neural behavior unpredictable. In order to simplify the analysis, the smallest network of two neurons was used. As was described in the previous chapters, STDP was the selected learning rule for network training. In order to model neural assemblies it is important to obtain realistic plasticity and neural behavior. The pair-based model of STDP represents two neurons connected recurrently and trained with the suitable STDP rule.

A feed-forward connection from a presynaptic cell to the postsynaptic side is independent from other similar connections and neighboring neurons. This feature called locality was described in part 2.2.4. However, the relation between neurons connected recurrently is controversial. Stability was the most important requirement of the trained network and unlimited growth of synaptic weights makes the network unstable. The stability of a network is due to the convergence of the synaptic weights to a steady value. The additive STDP rule is not capable of satisfying this need. Therefore, the first task was to modify the classical STDP rule in order to satisfy the mentioned need of recurrently connected neurons.

Various parameters affect synaptic weight change and neurons behave differently depending on how they are stimulated. Thus, the representation of the thalamic input was the second challenge encountered. According to the expanded Taylor equation of the spike-based Hebbian learning rule in part 2.2.4.2 both Hebbian and non-Hebbian learning can be considered as effective components of synaptic weight change. Therefore, the suggested model includes both Hebbian and non-Hebbian learning rules.

3.2 Dynamics of Cortical Neurons

Neural dynamics was simulated with MATLAB, using a simulation time-step of 1 millisecond. In order to simulate a network of cortical neurons, a small network of two excitatory neurons was selected. These two neurons are based on the Izhikevich model as described in section 2.1.3 and connected to each other recurrently as shown in Figure 3.1. Based on axonal length,
conduction delay was assigned for spikes traversing from a presynaptic cell to the postsynaptic side. In rabbits, Swadlow found axonal conduction delays of cortico-cortical connections to be between 1.2 and 19 ms. In Izhikevich’s polychronous groups, this evidence was used to set synaptic delays to random integers between 1 and 20 ms. When Izhikevich randomly generated neurons, the conduction delay of one connection could be small while the delay of its reciprocal connection could be large. The place where dendrite and axon meet, the synapse, may be distal from the neurons, so the conduction delay is large. The other connection may be created while the joint between axon and dendrite of neurons is close to the neurons and the delay is short. The Izhikevich neurons in this model are stimulated externally and the neural membrane potential changes at each time-step based on equation (3.1). An action potential is initiated when the membrane potential reaches a threshold of 30 mV. At this voltage apex of the spike, the membrane potential is reset to the value of $c$, as described in part 2.1.3.

$$v' = 0.04v^2 + 5v + 140 + u + I$$

$$u' = a(bv - u)$$

$v$ and $u$ represent membrane potential and membrane recovery. The intrinsic synaptic input is a consequence of the described non-Hebbian learning rule.

![schematic](image)

**Figure 3.1.** Two excitatory neurons connected recurrently are stimulated externally.

Spiking behavior of the neurons are controlled by parameters $a$, $b$, $c$. Parameters $d$, $a$ and $b$ regulate depolarization before the firing threshold and parameter $c$ and $d$ regulate repolarization after the spike occurs. Parameters $(b, c)$ are set for all neurons to $(0.2, -65)$ and parameters $(a, d)$ are set to $(0.02, 8)$ for excitatory neural activity. Synapses are updated at each time-step according to the implemented STDP learning rule. Synaptic weights $(s)$ of both synapses are initialized close to zero at 0.001 pS and the maximum synaptic conductance is 20 pS.

$$s = \max(0, \min(sm, 0.01 + s + sd))$$
Synaptic weights can never become negative or exceed the maximum conductance. Synaptic weight change during training is represented by $sd$. It is initially set to zero and can become positive or negative based on potentiation or depression during training. At each time step, the weight is calculated based on equation (3.3). The MATLAB code for the model is in the Appendix.

### 3.3 Learning Model

In chapter 2, different learning models were introduced which can be divided into the major categories of Hebbian and Non-Hebbian learning rules. Two aspects of Hebbian learning are firing frequency and time. Spike-based Hebbian learning and non-Hebbian learning contributed to the designated STDP learning rule used in this research. Their individual contributions are described in detail as follows.

#### 3.3.1 Spike-Based Hebbian Learning rule

The low time resolution of the rate-based learning model and higher precision of a 1 ms time step allowed the development of a Hebbian learning rule in terms of spike timing as well. The STDP model was developed with this in mind.

The first and most important step was defining a suitable STDP rule capable of overcoming the deficiencies of classical form of STDP. One of the problems with the classical STDP rule is its failure in reaching a steady state. According to described STDP models, additive and multiplicative, this problem can be solved by making the synaptic modification dependent to synaptic weight during training. Multiplicative rules control the convergence of the synaptic weight. The multiplicative model represents a stable unimodal of weight distribution, while the unstable bimodal weight distribution is the result of additive form (Section 2.2.5.4).

$$W(\Delta t) = \begin{cases} (w_{\text{max}} - w_{ji})a_+ \exp\left(\frac{\Delta t}{\tau_+}\right) & \text{if } \Delta t < 0 \\ -w_{ji}a_- \exp\left(\frac{\Delta t}{\tau_-}\right) & \text{if } \Delta t \geq 0 \end{cases} \quad (3.4)$$

The degree of potentiation and depression based on the multiplicative STDP is shown in the Figure 3.2. As was described, when the weight is close to the minimum value, the depression level is low and potentiation is high. When $W=0$, depression is zero and potentiation is at its maximum. When $W=20$, potentiation is zero and depression is at its maximum. In other words, the first and the last picture in Figure 3.2 show the hard bounds and the other pictures show the soft bounds. The level of depression and potentiation in the pictures, except the first and last one, depends on how close the synaptic weight is to the boundaries. Another influence on synaptic modification is the effect of the presynaptic spike on a postsynaptic
membrane potential. There are different opinions among neuroscientists on this issue. As mentioned earlier, different views are represented in the ‘nearest neighbor’ and ‘all to all’ frameworks. Choosing between these approaches depends on the needed accuracy of the computational results as compared to experimental observations and the model requirements.

![Figure 3.2: Multiplicative STDP](image)

Figure 3.2. Multiplicative STDP. The changes of synaptic EPDP are weight dependent. When $W=0$, the synaptic weight has its maximum potentiation and when $W=20$, depression is in its maximum value.

Some experimental data shows that the inter-spike interval can be longer than the connection delay between neurons. In this case, the outcomes of ‘nearest neighbor’ and ‘all to all’ STDP models are the same. Based on this, the ‘nearest neighbor’ method was chosen for calculating the effects of presynaptic spikes.

### 3.3.2 Non-Hebbian Learning Rule

Considering the detailed description of the non-Hebbian learning role on synaptic modification, this contribution was not neglected in our model. The contribution of the non-Hebbian model to synaptic modification is considered from two aspects.

According to the consensus on non-Hebbian learning, every presynaptic spike (e.g. $n_1$ in Figure 3.1) regardless of the firing of the postsynaptic side $n_2$, has a positive effect (EPSP) on the postsynaptic membrane potential. This theory is applicable while the presynaptic cell is excitatory. Alternatively, the firing of each neuron not only affects its own synaptic weights, but also contributes to synaptic formation in the opposing direction. Contrary to the first aspect of the non-Hebbian model, the presynaptic spike arrival affects the synaptic weight of
the opposite connection negatively. These first and second aspects can be called non-Hebbian potentiation and depression respectively, and will now be described.

a. Non-Hebbian Potentiation

In this project, each neuron can be depolarized in two ways, externally and internally. The variable $I$ represents the total transferred current to each neuron. Thalamocortical inputs are external currents that can stimulate neurons asynchronously or synchronously. In the following sections the stimulation process will be described in detail. The effect of each neuron on its neighbor starts when the spike arrives at the synapse. The excitatory postsynaptic potential (EPSP) and inhibitory postsynaptic potential (IPSP), the result of flow of positive and negative ions respectively, are interpreted as the internal currents. Since the analyzed neurons in this assertion are both excitatory, the positive effect of presynaptic spike arrival should be calculated. Ohm’s law represents simply the relation between the current flowing between two points and the potential difference between these two points.

According to equation 3.5 there is a direct relation between $I$ and $V$. The conductance of a membrane depends directly to the effects of all previously fired spikes. Equations 3.7 and 3.8 clarify this statement easily. Electrochemical driving force, $(V_f(t) - E_j)$, in cell membrane corresponds to the potential difference between two points of a conductor. When the cell membrane electrochemically is in balance, this value is zero, $V_f(t)$ represents the membrane potential and $E_j$ represents the reversal potential. As mentioned previously, the threshold of firing is $+30$ mV. In order to excite the postsynaptic cell, the reversal potential for $Na^+$ was given a larger value. $E_{Na^+} = 50$ mV has been chosen to find the excitatory potential$^{41}$.

$$I = V g$$

$$I_{Syn}(t) = g_{Syn}(t) (V_f(t) - E_j)$$

$$g_{SynE}(t) = \sum_j \left( \sum_{t_{kj} < t} S(w_{ji}) \exp \left( - \frac{(t - t_{kj})}{\tau_{SynE}} \right) \right)$$

where the function $S$ is as follows:

$$S(x) = \begin{cases} x & x \geq 0 \\ 0 & x = 0 \end{cases}$$

Equation (3.7) shows the integrated effect of inputs from different presynaptic cell $j$ to postsynaptic $i$ at time $t_{kj}$. Each spike arriving to neuron $i$ from neuron $j$ at time $t_{kj}$ increases the excitatory synaptic conductance or decrease inhibitory synaptic conductance. $g_E$ and $g_I$ represent the maximum increase in the excitatory and inhibitory synaptic conductance. $\tau_{SynE}$ and $\tau_{SynI}$ represent the decay time constants for the excitatory and inhibitory
conductance, respectively. The mentioned parameters get the following values, $g_E = g_I = 1 \text{ nS}$, $\tau_{\text{SynE}} = 5 \text{ ms}$, $\tau_{\text{SynI}} = 15 \text{ ms}$. Since this research is based on pair-wise connections, analyzing the cooperativity among the presynaptic inputs is impossible. In order to show the effect of cooperation in the non-Hebbian rule, the maximum conductance of excitatory synapse was assigned the value $g_E = 2 \times 10^{-2} \text{ S}$ [35].

**b. Non-Hebbian Depression**

One of the inadequacies of standard STDP is its incapability of forming recurrent connections. The STDP model is designated to modulate the synaptic weight of a feed forward connection. Based on the modified STDP rule, the pre- and postsynaptic side of a feed-forward connection are the only factors involved in synaptic modification. However, these two are necessary but not sufficient in a recurrent connection. Since, a recurrent synapse can be effected by the opposing one.

In recurrent networks, the concept of causality that has been described before can be seen in both connections. An increase of presynaptic firing has a negative effect on the opposing connection. Babadi presents its effect proportional to the synaptic weight and firing rate:

$$\frac{dw_{12}}{dt} = -B r_2 w_{12}$$  

$$B = \frac{A \tau_+ \tau_-}{\tau_m (V_{\text{th}} - V_r) (\tau_+ + \tau_-)}$$

When the firing of $n_1$ contributes to the firing of $n_2$, the synaptic weight $w_{12}$ of the connection from $n_1$ to $n_2$ increases. The cumulative EPSP increases the firing rate of $n_2$, which can cause the potentiation of $w_{21}$ and the depression of $w_{12}$.

$$\frac{dw_{12}}{dt} = F(w_{12}, \Delta t) - B r_2 w_{12} + \text{Cons.}$$  

The $\text{Cons}$ represents a constant 0.01 is the activity independent increase of synaptic weight, as used in the Izhikevich model. The synaptic effect of the first two learning rule components are summed linearly. The contribution of the positive non-Hebbian potentiation was described in the dynamics of the network as an intrinsic synaptic current. $\frac{dw_{12}}{dt}$ represents the weight change of a connection from $n_1$ to $n_2$. 


3.4 Thalamic Input

Thalamic input can be represented as many small EPSPs arriving to cortex synchronously or asynchronously and can be simulated by deterministic or stochastic means. Several different forms of input stimuli were explored, including fixed EPSPs with deterministic and stochastic stimulation, single sine modulated deterministic stimulation, fixed EPSPs with heterogeneous mean firing rates and dual sine modulated deterministic stimulation. The frequency and the magnitude of the external stimuli are tuned to fire neurons in frequency ranges of alpha, beta and gamma.

3.4.1 Constant Deterministic Stimulation

Neurons are excited periodically with a fixed interval and amplitude. The length of interval and input magnitude associates with the neural firing frequency that is biologically plausible. To analyze the effect of neural firing frequency on the stability of the synaptic weight, the external inputs were tuned to fire neurons in different frequency ranges.

3.4.2 Constant Stochastic Stimulation

To employ more biologically plausible stimulation, neurons were also excited by asynchronous input generated from a Poisson process. Experiments illustrate the irregularity in inter-spike interval of cortical neurons. If the independency of spikes are assumed, the randomly generated spike train would describe a Poisson process. To produce the spike train, a random number is generated at a fixed periodic interval. If the random number is less than 0.5, then the stimulation occurs.

3.4.3 Sine Modulated Deterministic Stimulation

To represent synchronous Thalamic input, the constant deterministic stimulation was amplitude modulated with the top part of a sin function, such that amplitudes are always positive or zero. The inter-spike intervals of the inputs are fixed as before. To simulate the synchronized population of stimulation neurons with a sine function, some steps were followed. Present a modified sine function with the frequency of $f$ Hz. The coefficient $2\pi f$ is the angular frequency which shows the numbers of oscillations in each time-step. Although the time-step of training was 1 ms, in the following formula $t$ was divided by 1000 to convert the time interval to seconds. Therefore, $f$ represents the number of oscillations which occur in a second.

$$s(t) = \max\left(\sin\left(\frac{2\pi ft}{1000}\right), 0\right)$$

(3.12)
Modulated Stimulation = Stimulation × \frac{8}{\pi} s(t)  \ (3.13)

Where \(s\) is the modulation and time \(t\) is in milliseconds shows the simulation modulated with a sine wave.

Two parameters, the periodic thalamic input stimulation interval and the input strength can be adjusted. Different values result in different firing patterns. The large values of these parameters result in multiple firings in each cycle of sine diagram. When the number of firings is greater than one, the firing intervals of each neuron are not regular. When the input intervals are 5 ms and the Stimulation is 15 mV, there are multiple firings in each cycle. The intervals between these firings are variable and unpredictable.

Figure 3.3. Sine modulated deterministic Stimuli

Figure 3.4. Tuning for a single spike per cycle of 10 Hz synchronous stimulation. (a) Multiple spikes before adjusting the EPSP amplitude. (b) Single spike per cycle after adjusting the EPSP amplitude.

Figure 3.4(a) shows the multiple firing of a neuron in each cycle. According to \(f\), these two parameters should be tuned to get a single firing in each cycle. The single spike is depicted in Figure 3.4 (b).

Once tuning is complete, an experiment with synchronous stimulation can be performed. To describe the method in detail, an example is presented. Figure 3.5 illustrates two neurons firing with the frequency of 34 Hz, giving them an inter-spike interval of almost 30 ms. Initially, when the weights are near zero, both neurons fire exactly at the peak of the sine function. The connection delay for \(n_1\) to \(n_2\) is 3 ms, and for \(n_2\) to \(n_1\) it is 17 ms. The synapse between \(n_2\) and \(n_1\) will grow when the delay from \(n_2\) to \(n_1\) is long enough to deliver a pre-
synaptic EPSP to the next cycle, within the STDP time window for weight growth. After repeated cycles, synaptic EPSPs will potentiate n1’s membrane, causing a negative phase shift of n1 (with respect to n2) in firing activity. The synapse between n1 and n2 will grow when the delay from n1 to n2, is short enough that the EPSPs from n1 will arrive at n2’s membrane before n2 fires. The weight growth and subsequent EPSPs can cause a negative phase shift of n2 (with respect to n1). According to the delay from n1 to n2 and the degree of n2’s phase drift, n1’s spike arrival might follow the n2’s firing and the synapse from n1 to n2 becomes depressed. Through STDP, the firings of n1 and n2 compete to reach an equilibrium. An equilibrium point, the synapses are stabilized.

Figure 3.5. The firing cycle in a recurrent connection between neurons n1 and n2, stimulated with Sine modulated stimuli. The dashed red line shows the phase drift of n1.

3.4.4 Heterogeneous Stimulation

In contrast with the previous methods, neurons are not stimulated with the same input. Each neuron is excited by deterministic stimuli with a specific inter-spike interval and strength. Therefore, the frequencies of neurons are different. The process of stimulation is the same as described methods in parts 3.4.2 and 3.4.3.
4 Simulation Results

Computational experiments were performed to determine the synaptic weight stability under various conditions. The two neuron network was stimulated separately with four different input sources. The inter-spike interval and the magnitude of stimuli affected the firing frequencies of neurons. Therefore, the neuronal firing rates can be analyzed as a parameter affecting the stability of the synaptic weights in three forms: Bidirectional, unidirectional and quiescent growth.

In each section that follows the synaptic modification of neurons were analyzed based on their firing frequencies. Alpha oscillations represent frequencies between the range of 8 to 13 Hz, beta oscillations covers the frequencies between 13 to 30 Hz and gamma oscillations are in the range of 30-100 Hz.

4.1 Constant Deterministic Stimulation

The first input source contains EPSPs with a fixed interval and magnitude. Synaptic modifications of neurons are analyzed under different circumstances. The input amplitude was tuned to make the neurons fire at different rates. In the first and second experiment, the neurons are firing in the ranges of alpha and gamma respectively.

4.1.1 Alpha Firing Rates

To generate alpha firing rates, e.g. 10Hz, the neurons are stimulated every 8 ms with amplitude of 20 mV. The recurrent synaptic weights were initialized to near zero, and as time passed as is depicted in Figure 4.1, the synaptic weight of the connection from $n_1$ to $n_2$ had unlimited growth and the synaptic weight in the opposing direction dropped after initially being potentiated.

![Figure 4.1. Synaptic EPSPs of neurons $n_1$ and $n_2$ with constant deterministic stimuli. The connection delay from $n_1$ to $n_2$ is 10ms and from $n_2$ to $n_1$ is 20ms. Firing frequencies are 10 Hz](image-url)
A negative drift of the spike timing of $n_2$ resulted in a decreasing inter-spike interval and a higher frequency. This continual shift was the reason behind the synaptic instability. Figure 4.2 (a) and (b) represent the cumulative changes of EPSPs. As is depicted in Figure 4.2 (a), the moving average of EPSPs on $n_1$ change between 0 and 1 mV and does not converge to a steady state. These unstable positive values result in the unstable synaptic weight. Figure 4.2(b) shows the cumulative EPSP changes of the connection from $n_2$ to $n_1$. The zero value of the synaptic weight change is due to the absence of Hebbian potentiation. Figures 4.2(c) and (d) illustrate the contribution of STDP and non-Hebbian learning rule on the synaptic modification. Figures 4.2 (e) and (f) zoom in on the effect of non-Hebbian on weight changes. The instability of synaptic EPSP changes results in the instability of the synaptic weight. It was clearly observed that having three forms of synaptic modification (bidirectional, unidirectional and no weight growth) was impossible. So the synaptic weight change is unpredictable and unstable.

![Figure 4.2](image)

**Figure 4.2.** (a, b) Mean changes of synaptic EPSP. (c, d) Contributions of Hebbian potentiation (orange), Hebbian depression (blue) and non-Hebbian depression (green) to weight dynamics with constant deterministic stimuli. (e, f) Non-Hebbian depression. The conduction delay from $n_1$ to $n_2$ is 10 ms (left column) and from $n_2$ to $n_1$ is 20 ms (right column) and the firing frequencies are 10 Hz.
4.1.2 Gamma Firing Rates

In this experiment, the interval and amplitude of the inputs are tuned to fire the neuron with a frequency of 40 Hz, in the gamma range.

Figure 4.3. Synaptic EPSPs of neurons \( n_1 \) and \( n_2 \) under constant deterministic inputs. The delay of the connection from \( n_1 \) to \( n_2 \) is 10 ms and the delay of the connection from \( n_2 \) to \( n_1 \) is 20 ms. Firing frequencies are 40 Hz.

Figure 4.3 shows the stable synaptic weights of connection from \( n_1 \) to \( n_2 \) with delay of 10 ms and the opposing direction with delay of 20 ms.

Figure 4.4. (a, b) Mean changes of synaptic EPSP. (c, d) Contributions of Hebbian potentiation (orange), Hebbian depression (blue) and non-Hebbian depression (green) to weight dynamics with constant deterministic stimuli. (e, f) Non-Hebbian depression. The conduction delay from \( n_1 \) to \( n_2 \) is 10 ms (left column) and from \( n_2 \) to \( n_1 \) is 20 ms (right column) and the firing frequencies are 40 Hz.
The mean inter-spike intervals between neurons are 25 ms. Thus, according to conduction delays, the arrival of the presynaptic spike before postsynaptic spike is more probable than the reverse order. Therefore, during the early stages of training, potentiation overcomes depression and the synaptic weight starts growing. The ratio of depression to potentiation is less than one. The arrival of presynaptic spikes induces EPSPs on the postsynaptic membrane. The total changes of EPSPs result in negative drift of neuronal firing and an increase in firing frequency. The potentiation decreases and depression increases until the system reaches an equilibrium point. When the depression and potentiation are in balance, synaptic weight starts converging to a steady state. Figure 4.4 (a) and (b) illustrate the cumulative synaptic EPSP change. Regarding Figures (c) and (d), the mean depression is in balance with the mean potentiation. Therefore, the ratio of depression to potentiation converges to one and the cumulative change converges to zero, as depicted in figures (a) and (b). Figure 4.1 and Figure 4.3 illustrate the weight change of neurons with synaptic conduction delays of 10 and 20 ms but with different firing rates. The comparison of these two cases led me to conclude that the STDP rule contributes to the stability of neurons with a high firing rate.

4.2 Constant Stochastic Stimulation

Actual thalamic inputs are a combination of synchronous and asynchronous spikes. Asynchronous stimuli are generated in a stochastic process as described in chapter 3, with the use of a Poisson process. The probability of stimulation and the magnitude of inputs determine the firing frequencies of neurons. The inputs’ properties were tuned to fire neurons in the frequency ranges of alpha and gamma oscillations.

4.2.1 Alpha Firing Rates

In this section, the mean firing frequencies of neurons are 10 Hz. Figure 4.5 depicts the synaptic EPSPs of two connections. The delay of the connection from \( n_1 \) to \( n_2 \) is 10 ms and the opposing direction is 20 ms. As Figure 4.5 shows, the synapses of both connections are stabilized and converge to steady states. Figure 4.6 (a) and (b) illustrate the fluctuations of total changes of synaptic EPSP. The stability of the cumulative changes is the result of the balance between potentiation and depression. Since both neurons are excited randomly and the mean inter-spike interval between the inputs is 100 ms, the occurrence of the presynaptic spike arrival before the postsynaptic firing is more probable than the reverse order. Therefore, the synaptic weights were potentiated and grew.
Figure 4.5. Synaptic EPSPs of neurons $n_1$ and $n_2$ with constant stochastic stimuli. The delay of the connection from $n_1$ to $n_2$ is 10 ms and the delay of the connection from $n_2$ to $n_1$ is 20 ms. Firing frequencies are 10 Hz.

The cumulative EPSP causes the neurons’ firing to drift negatively in phase. Therefore, the inter-spike intervals decrease and firing frequency increases. During training, the depression increases. The system moves toward equilibrium when potentiation and depression are in balance. Figure 4.6 (a) and (b) confirm the stability of the synaptic weights. The total changes of synaptic EPSP in both membranes converge to zero.

Figure 4.6. (a, b) Mean changes of synaptic EPSP. (c, d) Contributions of Hebbian potentiation (orange), Hebbian depression (blue) and non-Hebbian depression (green) to weight dynamics with constant deterministic stimuli. (e, f) Non-Hebbian depression. The conduction delay from $n_1$ to $n_2$ is 10 ms (left column) and from $n_2$ to $n_1$ is 20 ms (right column) and the firing frequencies are 10 Hz.
4.2.2 Gamma Firing Rates

Based on the results observed in section 4.1.2, it was worth analyzing the synaptic modifications of neurons, which were excited stochastically while the firing rate was in the gamma range.

Figure 4.7. Synaptic EPSPs of neurons $n_1$ and $n_2$ stimulated by asynchronous random inputs. The delay of the connection from $n_1$ to $n_2$ is 10ms and the delay of the connection from $n_2$ to $n_1$ is 20ms. Firing frequencies are 37 Hz.

The firing rates of neurons in Figure 4.7 are 37 Hz. The synaptic weights of neurons converge to a steady state after being repetitively stimulated asynchronously.

Figure 4.8. (a, b) Mean changes of synaptic EPSP. (c, d) Contributions of Hebbian potentiation (orange), Hebbian depression (blue) and non-Hebbian depression (green) to weight dynamics with constant deterministic stimuli. (e, f) Non-Hebbian depression. The conduction delay from $n_1$ to $n_2$ is 10 ms (left column) and from $n_2$ to $n_1$ is 20 ms (right column) and the firing frequencies are 37 Hz.
Figure 4.8 shows the contribution of STDP to the synaptic stability. The mean value of potentiation equals to the mean depression value, consequently the synaptic weight changes converge to zero.

4.3 Sine Modulated Deterministic Stimulation

Three forms of stable synaptic modification in a recurrent connection were introduced in Chapter 3. So far, only stable bidirectional synaptic growth has been observed. To create three forms of recurrent connection, neurons are stimulated with sine modulated deterministic stimuli. The inputs were tuned to fire both neurons with frequencies of 10, 30, 40 and 50 Hz.

4.3.1 Firing Rate of 10 Hz

Figure 4.9 depicts the synaptic weights of two neurons connected recurrently. The interval between the spike of neurons firing with the rate of 10 Hz is 100 ms. The maximum conduction delay is 20 ms. If the STDP time window was 20 ms, neither of the presynaptic spike arrivals would be in the temporal window of its postsynaptic side. Therefore, neurons firing with the frequencies of 10 Hz do not grow, regardless of the conduction delays of the connections between them.

![Figure 4.9. Synaptic EPSPs of neurons $n_1$ and $n_2$ with sine modulated inputs. The delay of the connection from $n_1$ to $n_2$ is 1ms and the delay of the connection from $n_2$ to $n_1$ is 10ms. Firing frequencies are 10 Hz.](image)

The Hebbian and Non-Hebbian depression are the only factors affecting the synaptic weight. The absence of potentiation results in the quiescent synaptic weights. To produce three forms of synaptic weight structure, the inputs were tuned to fire neurons at higher rates.

4.3.2 Firing Rate of 30 Hz

a. Bidirectional Synaptic Weights

Figure 4.11 shows stable bidirectional synaptic weights between $n_1$ and $n_2$. The delay of the connection from $n_1$ to $n_2$ is 15 ms and in opposing direction is 20 ms. The interval between the neuronal spikes is 33ms. Regarding to the conduction delays, the arrival of the presynaptic spike occurs before the postsynaptic firing. So, both synaptic weights are potentiated.
Figure 4.10. Synaptic EPSPs of neurons $n_1$ and $n_2$. The delay of the connection from $n_1$ to $n_2$ is 15 ms and the delay of the connection from $n_2$ to $n_1$ is 20 ms. Firing frequencies are 30 Hz.

According to Figure 4.11 (a) and (b), the synaptic weight changes converge to zero. During the early stages of the stimulation, the potentiation overcomes depression, but during the following time-steps, depression increases and potentiation decreases. After 100 seconds, the depression and potentiation are in balance and the synaptic weights reach stability.

Figure 4.11. (a, b) Mean changes of synaptic EPSP. (c, d) Contributions of Hebbian potentiation (orange), Hebbian depression (blue) and non-Hebbian depression (green) to weight dynamics with constant deterministic stimuli. (e, f) Non-Hebbian depression. The conduction delay from $n_1$ to $n_2$ is 15 ms (left column) and from $n_2$ to $n_1$ is 20 ms (right column) and the firing frequencies are 30 Hz.
b. Unidirectional Synaptic Weights

Figure 4.13 shows the synaptic modification of $n_1$ and $n_2$. The delay of the connection from $n_1$ to $n_2$ is 11 ms and the opposing direction is 18 ms. The connection delay between $n_2$ and $n_1$ is long enough to potentiate the membrane during the next firing cycle.

![Graph showing synaptic modification](image)

Figure 4.13. Synaptic EPSPs of neurons $n_1$ and $n_2$. The delay of the connection from $n_1$ to $n_2$ is 11 ms and the delay of the connection from $n_2$ to $n_1$ is 18 ms. Firing rate is 30 Hz.

The arrival of $n_2$ does not occur in the temporal window of $n_1$. Therefore, only the synaptic weight from $n_1$ to $n_2$ grows and the other connection is quiescent.

![Graph showing synaptic changes](image)

Figure 4.13(a, b) Mean changes of synaptic EPSP. (c, d) Contributions of Hebbian potentiation (orange), Hebbian depression (blue) and non-Hebbian depression (green) to weight dynamics with constant deterministic stimuli. (e, f) Non-Hebbian depression. The conduction delay from $n_1$ to $n_2$ is 11 ms (left column) and from $n_2$ to $n_1$ is 18 ms (right column) and the firing frequencies are 30 Hz.
The synaptic weight change in Figure 4.13 (a) converges to zero. Figure (c) shows the balance between depression and potentiation. In contrast to figure (a), the cumulative EPSP changes drop to negative values in figure (b). The absence of the STDP contribution is the cause of a quiescent connection between $n_2$ and $n_1$. The only active factor effecting the synaptic weight of this connection is the non-Hebbian depression. Regarding of the firing rate and conduction delays, a recurrent connection forms in a stable feed-forward manner.

**a. No Weight Growth**

The choice of two short conduction delays make both synapses quiescent as depicted in Figure 4.14. The conduction delays of the synapses are 1ms in both directions. According to the firing rate and conduction delays, the arrival of a presynaptic spike before a postsynaptic firing is not possible and subsequently no growth of synaptic weight is possible. The neural membrane is only subject to non-Hebbian effects. The absence of Hebbian potentiation causes the synaptic weights to be quiescent.

![Figure 4.14. Synaptic EPSP of neurons $n_1$ and $n_2$ with sine modulated stimuli. The delays from $n_1$ to $n_2$ and the opposing direction is 1 ms. The firing frequencies of neurons are 30 Hz.](image)

**4.3.3 Firing Frequency of 40 Hz**

**a. Bidirectional Synaptic Weights**

As Figure 4.15 depicts, a conduction delay of 15 ms between $n_2$ and $n_1$ is long enough to arrive before the spike of the $n_1$ during the next synchronous cycle, which allows the synaptic weight start growing from the start of training. Alternatively, due to the short connection delay from $n_1$ to $n_2$, the synaptic weight did not potentiate for the first 80 ms.

![Figure 4.15. Synaptic EPSPs of neurons $n_1$ and $n_2$ with sine modulated stimuli. The delay of the connection from $n_1$ to $n_2$ is 4ms and the delay of the connection from $n_2$ to $n_1$ is 15ms.](image)
The cumulative EPSP causes \( n_1 \) to drift negatively in phase while keeping the firing frequency fixed. These drifts cause the arrival of \( n_1 \)'s spike to precede the \( n_2 \) spike. Consequently, the synaptic weight of the connection from \( n_1 \) to \( n_2 \) starts increasing. Figure 4.16 show the stability of the synaptic weight change and the balance between depression and potentiation.

![Figures 4.16](image)

Figure 4.16. (a, b) Mean changes of synaptic EPSP. (c, d) Contributions of Hebbian potentiation (orange), Hebbian depression (blue) and non-Hebbian depression (green) to weight dynamics with constant deterministic stimuli. (e, f) Non-Hebbian depression. The conduction delay from \( n_1 \) to \( n_2 \) is 4 ms (left column) and from \( n_2 \) to \( n_1 \) is 15 ms (right column) and the firing frequencies are 40 Hz.

**b. No Weight Growth**

The other form of synaptic modification emerged when both of the connection delays are too short to allow potentiation of each other in the next cycle. As a result, neither of the synapses grows. Figure 4.17 shows the quiescent synaptic weights while the delay from \( n_1 \) to \( n_2 \) is 1 ms and the delay of the reciprocal connection is 3 ms. The absence of Hebbian potentiation causes the synaptic weight not to grow. There are many connections with short delays in mini-columns that are expected to be formed strongly. According to achieved results, the neuronal firing frequency affects the synaptic growth of short connections.
As the figures illustrate, bidirectional and quiescent weight growth are two possible forms of synaptic modifications of neurons firing rate of 40 Hz.

### 4.3.4 Firing Frequency of 50 Hz

Tuning the inputs to get the higher neural firing rate resulted in some changes in output. In part 4.3.2 three forms of synaptic growth, bidirectional, unidirectional and no weight growth were observed. While in part 4.3.3 inputs were tuned to cause neurons to fire with frequencies of 40 Hz, the forms of synaptic modification were reduced to recurrent and no weight growth. The difference between these two experiments motivated me to increase the firing rate of neurons to 50 Hz by tuning the external stimuli.

Since the STDP time window is ±20 ms, the arrival of any spike with any delay is in the temporal window of its postsynaptic side. Therefore, delays do not play a role in synaptic modification. Bidirectional synaptic modification is the only way that synapses behave. The contribution of STDP is the same as describe in previous sections. As is depicted Figure 4.19 in, the balance between the depression and potentiation and the convergence of the total changes of EPSP to zero verify the stability of both connections. Regarding the experimental results, the firing drift is limited to the cycle in which the neuron fires, so firing rate is fixed. Equation 4.1 shows the relation between the delay and the neural firing frequency.
Figure 4.19. (a, b) Mean changes of synaptic EPSP. (c, d) Contributions of Hebbian potentiation (orange), Hebbian depression (blue) and non-Hebbian depression (green) to weight dynamics with constant deterministic stimuli. (e, f) Non-Hebbian depression. The conduction delay from $n_1$ to $n_2$ is 4 ms (left column) and from $n_2$ to $n_1$ is 15 ms (right column) and the firing rate are 50 Hz.

Two functions are presented in equation 4.1. $P_1$ symbolizes the synaptic growth of neuron. The number 1 represents the potentiation of the connection from $n_1$ to $n_2$ and 0 shows the silent synapse. Mathematically, numbers 1 and 0 represent the situation on which the function is true or false respectively. It is the function of $d_1$. The synaptic growth depends on $f$: neural firing frequency, $\tau$: STDP time window and $d$: conduction delay. $P_2$ symbolizes the synaptic growth of connection from $n_2$ to $n_1$. The output of $P_2$ corresponds to two arguments, $d_1$ and $d_2$. Occurrence of $P_1$ is a necessary condition for $P_2$ but is not sufficient. When the condition $p_2$ is met, function $P_2$ gets the true value. Alternatively, synaptic growth of second neuron is due to the synaptic growth of the first one and its own conduction delay.

\[
P_1(d_{12}) = \begin{cases} 
1 & \text{if } T - \tau_s \leq d_{12} \leq T \\
0 & \text{otherwise}
\end{cases}
\]

\[
p_2(d_{21}) = \begin{cases} 
1 & \text{if } 0 \leq d_{21} < T - d_{12} \\
0 & \text{otherwise}
\end{cases}
\]

\[
P_2(d_{12}, d_{21}) = P_1(d_{12}) \land p_2(d_{21})
\]
The results show that if the conduction delay of the neuron drifting negatively would be less than $T - d_{12}$, its synaptic weight start growing. $T$ is the reciprocal of the firing frequency. Function $P_2$ can be defined by a Boolean conjunction, shown by $\land$.

### 4.4 Heterogeneous Stimulation

In the previous sections, the stimuli were tuned to fire neurons in the same range of frequency. As an example, the frequencies of neurons were in the range of gamma oscillation. Relations between the firing frequency, conduction delay, and STDP affecting the synaptic modification is determined. A question arose to whether it is possible to find such a relation for neurons firing at different rates. To answer this question, the inputs were tuned to fire neurons with rates in two different ranges of alpha-beta and alpha-gamma.

#### 4.4.1 Dual Constant Stimuli

##### 4.4.1.1 Alpha-Beta Firing Rates

In this part, neurons were excited with two sources of inputs. Both sources stimulated neurons through the constant deterministic inputs.

![Figure 4.20. Synaptic EPSPs of neurons $n_1$ and $n_2$ with heterogeneous stimuli. The delay of the connection from $n_1$ to $n_2$ is 16ms and the delay of the opposing direction is 10ms. The firing rate of neurons are 9.6 and 14.3 Hz respectively.](image)

Neurons firing at different rates cause them to be in different firing phases. As is depicted in Figure 4.20, neither of the synaptic weights are stable. Two connections with delays 16ms and 10 ms are under the excitation of the heterogeneous inputs. n1 is firing with the frequency of 9.6 Hz and the firing frequency of $n_2$ is 14.3Hz. Figure 4.21 (a) and (b) explain the instability of synaptic weights. The cumulative changes of small synaptic EPSPs are instable. The drop of the total changes in figure (a) to negative values represents the overcome of depression to potentiation. Figure (b) represents the positive value of the synaptic weight change resulting in the saturation of the synaptic weight. Figure 4.21(c) describes the drop of the synaptic EPSP of the connection from $n_1$ to $n_2$. The absence of Hebbian potentiation and the effect of non-Hebbian model cause the drop of cumulative changes to negative values. As is depicted in Figure 4.22, $n_1$ starts firing with the frequency...
of 9.6 Hz and the firing frequency of \( n_2 \) is 14.3 Hz. Neuron \( n_2 \) has the higher firing frequency, so the number of firings of \( n_2 \) is greater than \( n_1 \) in a certain period. A larger number of firings results in higher cumulative EPSPs. The EPSP from \( n_2 \)'s spike onto \( n_1 \)'s membrane is stronger than the effect of \( n_1 \)'s spike on \( n_2 \)'s membrane.

Therefore, the firing frequency of \( n_1 \) increases to reach the frequency of \( n_2 \). Then neurons start firing synchronously.
4.4.1.1 Alpha-Gamma Firing Rates

The Figure 4.23 shows the stable synaptic weights of the recurrently connected neurons. The delay of the connection from $n_1$ to $n_2$ and opposing direction are 20 ms. The initial firing rate of $n_1$ and $n_2$ are 15.7 Hz and 38.8 Hz, respectively. The contribution of Hebbian potentiation and depression clearly can be observed in Figure 4.24.

Figure 4.23. Synaptic EPSPs of neurons $n_1$ and $n_2$ with heterogeneous stimuli. The delay of the connection from $n_1$ to $n_2$ is 20 ms and from $n_2$ to $n_1$ is 20 ms. The firing rates of neurons are 15.7 and 38.8 Hz respectively.

The mean value of the depression and potentiation are in balance. Therefore, the synaptic weight change converges to zero.

Figure 4.24. (a, b) Mean changes of synaptic EPSP. (c, d) Contributions of Hebbian potentiation (orange), Hebbian depression (blue) and non-Hebbian depression (green) to weight dynamics with constant deterministic stimuli. (e, f) Non-Hebbian depression. The conduction delay from $n_1$ to $n_2$ is 20 ms (left column) and from $n_2$ to $n_1$ is 20 ms (right column). The firing rates of $n_1$ and $n_2$ are 38.8 Hz and 15.7 Hz respectively.
The fluctuations in Hebbian potentiation and depression and non-Hebbian depression are the result of different firing rate and subsequently different phases of firing at the same time. Figure 4.25 shows the synchronization of the neurons firing with different frequencies. The projection of $n_1$ on $n_2$ increases the firing frequency of $n_2$. Neurons start firing synchronously after 150 seconds which corresponds to the time the synaptic weights in Figure 4.23 converge to the steady level.

![Figure 4.25. Heterogeneous stimuli. Neurons $n_1$ and $n_2$ initially are firing with the frequencies of 38.8 Hz and 15.7 Hz respectively. The frequency of $n_1$ reaches the frequency of the $n_2$.](image)

4.4.2 Dual Sine Modulated Stimuli

Figure 4.26 shows two sine modulated stimuli with different amplitudes and firing rates. Different firing rate cause two neurons to be on different phases at a single time.

In this case, the spike arrival of presynaptic cell may precede or follow the postsynaptic cell. It is clear that the potentiation of the connection from a neuron with a higher frequency to a neuron with a lower one is more probable than the facilitation of the opposing connection.

![Figure 4.26. Different external stimuli. The top diagram shows the external stimuli firing rate of 30 Hz and the bottom oscillation shows the stimuli firing rate of 40 Hz.](image)

The presynaptic spike of the neuron with higher firing rate precedes the postsynaptic spike more often, due to the larger number of firings. Thus, the cumulative EPSP of neuron with lower firing rate is large enough to increase the firing rate. Induction of a strong EPSP causes the negative drifts in phase. The neuron with the lower firing rate increases its rate to the neuron with higher firing rate. Then, the neurons are synchronized in a temporal window.
4.4.2.1 Alpha-Beta Firing Rates

Figure 4.27 depicts the synaptic EPSP of two neurons. The delay of the connection from \( n_1 \) to \( n_2 \) is 10 ms and opposing direction is 20 ms.

![Synaptic EPSPs of neurons](image)

Figure 4.27. Synaptic EPSPs of neurons \( n_1 \) and \( n_2 \) with heterogeneous stimuli. The delay of the connection from \( n_1 \) to \( n_2 \) is 10 ms and the delay of the connection from \( n_2 \) to \( n_1 \) is 20 ms. Firing rate are 10 and 17 Hz respectively.

The initial firing rate of \( n_1 \) and \( n_2 \) are 10 Hz and 17 Hz respectively. According to the ratio of T1 (T is the reciprocal of firing frequency) to T2, the spike arrival of \( n_1 \) precedes \( n_2 \)'s spike and then the synaptic weight from \( n_1 \) to \( n_2 \) is potentiated. During the time the frequency of \( n_2 \) increases and comes close to the frequency of \( n_1 \). Therefore, the probability of depression of \( n_2 \) is increasing as well. The convergence of synaptic weight of a connection from \( n_2 \) to \( n_1 \) from 150 seconds to 200 seconds justifies this happening.

The mean firing rate of \( n_1 \) during the last 50 seconds is almost 17 Hz and neurons \( n_1 \) and \( n_2 \) are firing synchronously with a phase shift. While the firing frequency of \( n_1 \) approaches to the frequency of \( n_2 \), the firings of \( n_1 \) shift earlier and closer to firings of \( n_2 \) but do not reach them.

Although, the STDP rule causes the firing of neurons be synchronized, the process of convergence of the lower firing frequency to the higher frequency is so long and the synapses are subject to many different changes. So, both synaptic weights are unstable. The instability in the synaptic weight change can be observed clearly in the Figure 4.28 (a) and (b). The cumulative changes of synaptic EPSP of \( n_1 \) dropping to negative values justifies the drop of the synaptic weight of the connection from \( n_1 \) to \( n_2 \) to zero in Figure 4.27. Large positive the synaptic weight change of \( n_2 \) Figure 4.28 (b) cause the synaptic weight of the connection between \( n_2 \) and \( n_1 \) to have unlimited growth up to the boundaries. The frequencies of neurons \( n_1 \) and \( n_2 \) are calculated with the aid of a moving average within a 700 ms window. According to the Figure 4.29, the neuron with the higher firing rate excites the one with lower rate, until the rate of \( n_1 \) reaches the rate of \( n_2 \).
Figure 4.28. (a, b) Mean changes of synaptic EPSP. (c, d) Contributions of Hebbian potentiation (orange), Hebbian depression (blue) and non-Hebbian depression (green) to weight dynamics with constant deterministic stimuli. (e, f) Non-Hebbian depression. The conduction delay from $n_1$ to $n_2$ is 10 ms (left column) and from $n_2$ to $n_1$ is 20 ms (right column). The neural firing rate are 10 and 17 Hz, respectively.

Since the firing rate of $n_2$ is larger than the rate of $n_1$, the arrival of $n_2$’s spike before $n_1$’s firing is more probable than the reverse order. Regarding the temporal window ($\pm 20$ ms), conduction delays (maximum 20 ms) and firing rate (10Hz and 17 Hz) the synaptic weight does not converge to steady state.

Figure 4.29. Dual sine modulated stimuli. The neural firing rates of $n_1$ and $n_2$ are 10 and 17 Hz, respectively.
4.4.2.2 Alpha-Gamma Firing Rates

Here, neurons $n_1$ and $n_2$ are firing at 10 Hz and 40 Hz respectively. The conduction delays of recurrent connections are 10 ms.

![Figure 4.30](image)

**Figure 4.30.** Synaptic EPSPs of neurons $n_1$ and $n_2$ with heterogeneous stimuli. The delay of the connection from $n_1$ to $n_2$ is 10 ms and the delay of the connection from $n_2$ to $n_1$ is 10 ms. Firing rates are 10 Hz and 40 Hz respectively.

The described firing drifts and increase of frequency of the neuron with lower firing rate in section 4.4.2.1 occur between neurons firing in gamma. The only difference between these two cases is the speed of firing frequency changes and its convergence to a steady state.

![Figure 4.31](image)

**Figure 4.31.** (a, b) Mean changes of synaptic EPSP. (c, d) Contributions of Hebbian potentiation (orange), Hebbian depression (blue) and non-Hebbian depression (green) to weight dynamics with constant deterministic stimuli. (e, f) Non-Hebbian depression. The delay from $n_1$ to $n_2$ is 10 ms (a) and the delay of the opposite connection is 20 ms (b). The neural firing rate are 10 and 40 Hz respectively.
Figure 4.30 illustrates the synaptic stability of neurons firing with different frequencies. As is depicted in Figure 4.31 (a) and (b), the convergence of the synaptic weight change to zero is due to the balance between depression and potentiation. The firing frequency of n1 is four times more than the frequency of n2. Therefore, the number of firings and the probability of being potentiated are four times more. In the other respect, the durations of one cycle in the firing of n1 and n2 are 100ms and 25 ms respectively. The delay from n1 to n2 is greater than the interval between the firing of the n1 and n2, so the spike of the n1 arrives after the firing of n2. Therefore, the synaptic weight of connection from n1 to n2 does not grow from the start of stimulation. According to the delay of the connection from n2 to n1, membrane of n1 is projected by n2 spikes and its firing frequency increases. Figure 4.32 illustrates the synchrony of n1 and n2 at 100 seconds.

This time corresponds to the time the synaptic weights converge to steady states in Figure 4.30. Since the frequencies of both neurons are high and subsequently the periods are short both neurons project each other. As a result the frequencies of both neurons increases while firing synchronously.

![Figure 4.32. Dual sine modulated stimuli. The firing rate of n1 and n2 are 10 and 40 Hz, respectively.](image)
5 Discussion

The experimental results showed that some forms of stimuli preserved the neural firing rates while other stimuli did not. Among the described stimulation methods, the firing rate changed during training on all neurons except in the case of “sine modulated deterministic stimuli”. As was mentioned in previous chapter, the input was tuned to fire neurons in the alpha, beta and gamma frequency ranges before learning started. Analysis was then done to determine how these firing rates changed during and after learning.

5.1 Variable Firing Frequencies

Experiments revealed that spiking neurons with constant deterministic, stochastic or heterogeneous stimuli start drifting negatively in phase to a higher frequency, even when presynaptic spike arrival occurs after most spikes. Figure 5.1 (a) and (b) show the firing rate of neurons excited with stochastic stimuli while firing in the ranges of gamma and alpha, respectively. The negatively drifts of neural firings in both experiments result in increasing of firing frequencies. Initially, the ratio of the depression to potentiation is less than one, but during the time, it converges to one. The balance between depression and potentiation makes the firing frequencies stabilized. The stabilized firing frequencies in Figure 5.1 verify the stability of synaptic weights in Figure 5.2.

![Figure 5.1](image)

*Figure 5.1. The firing frequencies of spiking neurons with stochastic stimuli. a. The neurons are firing in the gamma range. b. Neurons are firing in the alpha range.*

In the case of heterogeneous stimulation, the neurons fire at different rates. Figure 4.29 and Figure 4.32 show neurons in the range of alpha-beta and alpha-gamma firing rates, respectively. Both experiments illustrate the synchronization of the neuron pair, which were initially firing at different rates. The synapses in the first experiment are unstable while the synaptic weights of both connections in the second experiment converge to a steady state.
5.2 Fixed Firing Frequencies

Figure 5.3 shows the relative stability of neuronal firing frequencies. In contrast with the constant stimulation, sine wave modulation controls the firing drift. Under certain condition, the length of conduction delay, the arrival of the presynaptic spike occurs before the postsynaptic firing. The accumulation of excitatory postsynaptic potential results in the negatively drifting of firing. The neuron is drifting to the earlier phases while the firing frequency is fixed.

If the initial phases of firing are different, the neuron with the earlier phases always precedes the spike of the other neuron. Neurons with different delays and firing frequencies have been analyzed. Results demonstrate the relation between these two factors. The relation is presented in the following equation.

\[
P_1(d_{12}) = \begin{cases} 
1 & T - \tau_4 \leq d_{12} \leq T \\
0 & \text{otherwise}
\end{cases}
\]

\[
p_2(d_{21}) = \begin{cases} 
1 & 0 \leq d_{21} < T - d_{12} \\
0 & \text{otherwise}
\end{cases}
\]

\[
P_2(d_{12}, d_{21}) = p_1(d_{12}) \land p_2(d_{21})
\]

(5.1)

Table 5.1 describes different conditions of function in equation (5.1).
Regarding to the achieved results by Swadlow, the axonal conduction delays of cortico-cortical connections are between 1.2 and 19 ms. So, the delays are integers between 1 and 20 ms. The relation between the firing frequency, delay and the type of external stimuli determine how synaptic weights grow.

Neurons firing with frequency less than 25 Hz could not be potentiated regardless of the conduction delay. Three forms of synaptic modification can be observed for neurons with firing frequencies between 25 and 36 Hz. One fourth of cycles of frequencies $36 < f < 50$ are greater than the upper boundary of $s_2, T − d_1$. Therefore, unidirectional modification does not form. Finally, the STDP temporal window equals to the reciprocal of firing frequency, $T$. Consequently, spike arrival of any synapse with any delay happen within the time window when the neurons’ frequencies are greater than 50 Hz. So, the bidirectional synaptic modification is the only form of synaptic modification. The following table summarized the described results. It should be mentioned that the following table presents the condition that synapses are in stability. Obviously, there are some cases that synaptic weights of neurons firing with alpha frequency growing recurrently, but the system is not stable.

<table>
<thead>
<tr>
<th>$P_1$</th>
<th>$P_2$</th>
<th>Bidirectional</th>
<th>Unidirectional</th>
<th>Quiescent</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>×</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>×</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>×</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5.1. The Boolean conjunction between two independent conditions.

<table>
<thead>
<tr>
<th>Firing Frequency (Hz)</th>
<th>Synchronous</th>
<th>Asynchronous</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Bidirectional</td>
<td>Unidirectional</td>
</tr>
<tr>
<td>$f &lt; 25$</td>
<td>×</td>
<td>×</td>
</tr>
<tr>
<td>$25 \leq f \leq 36$</td>
<td>×</td>
<td>×</td>
</tr>
<tr>
<td>$36 &lt; f &lt; 50$</td>
<td>×</td>
<td>×</td>
</tr>
<tr>
<td>$f \geq 50$</td>
<td>×</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.2. The relation between the firing frequency of a neuron and the synaptic modification.

Briefly speaking, a high firing rate represents a powerful parameter to control the synaptic modification. The type of the external stimuli tuned to fire neurons with high frequencies does not affect the synaptic modification. The synapses between these neurons are stable and grow recurrently regardless of the type of the external inputs, while this issue is imperative for neurons stimulated with low firing rates. Neurons with the higher firing frequencies converge to the steady state in a shorter time than the time is required for neurons with low firing frequencies. The convergence of synaptic weight to a steady point is the result of
balance between the depression and potentiation. Among the describe methods, the sine modulated stimuli are capable of creating three forms of stable synaptic modification. The stable bidirectional synaptic growth is the common output of the other methods.

The results show that STDP is capable of tuning the timing of the firings while neurons are firing synchronous or asynchronously with respect to their firing frequencies.
6 Conclusion and Future Work

As Cajal hypothesized, the storage of external information in the brain is due to the plasticity of the synapses. Hebbian learning suggests that activity dependent synaptic modification is based on correlated firing frequency. Limitations of the rate-based Hebbian learning rule relating to neural activities on the millisecond time scale was the basis for the emergence of the spike-based Hebbian learning rule, and the STDP rule was derived from this temporal aspect of Hebbian learning. The shortcomings of classical STDP in achieving synaptic stability inspired me to present a modified learning model capable of storing information in a pair of recurrently connected neurons. The study of the combination of STDP and non-Hebbian learning with different input stimulation produced some conclusive results.

In the simulations, external input sources, neural firing frequency and conduction delay were three parameters that affected synaptic stability. The degree of randomness of input, whether deterministic or stochastic, was not a significant factor when the fixed amplitude input stimulation was tuned to fire neurons at frequencies in the range of gamma oscillations. Regardless of the type of the external stimuli, the synaptic weights of these neurons were recurrently stable. The contributions of STDP and non-Hebbian learning to synaptic plasticity were verified with the emergence of three modified synaptic structures in a recurrently connected pair of neurons. Recurrent synaptic modification, feed-forward growth of synaptic weight and no weight growth were stable synaptic structures of stored information.

STDP is capable of tuning the induction of EPSP on the postsynaptic membrane through the regulation of the firing time of the neurons firing synchronously or asynchronously. The adaption occurs with negative phase drifts of the firing time while both keeping the firing frequencies of synchronized neurons intact and synchronizing the polysynchronized neurons. In both approaches the recurrent connections are stabilized and the storage of encoded information occurs as well. The stability of the synapse is due to the balance of potentiation and depression. STDP contributes to the formation of a group of neurons with strong connections firing synchronously in a temporal window.

In order to reduce the complexity of the project some assumptions and limitations have been applied. Considering a pair of neurons with recurrent connections rather than a network with large number of neurons imposed some restrictions on the research. In this study, the network is composed of excitatory neurons and the role of inhibitory ones was ignored.
The analysis of STDP in a pair of neurons can be a basis for investigating behavior of a larger network of neurons. As was mentioned, analyzing synaptic dynamics and structure are necessary for addressing cell assembly formation. Therefore, an extension of this work could be to study the formation of multiple cell assemblies with the presence of recurrent connections, in a larger network. This would include applying more complex and multiple input patterns. Applying the designed model to a larger network with both excitatory and inhibitory neurons makes the network more similar to the nervous system. The biological level of the current model is the level of the ‘integrate and fire’ model. In order to increase the level of realism of the simulation, such models as Hodgkin–Huxley with more detailed biological characteristics could be applied.
7 Appendix

The MATLAB code simulating the network is presented in this section. The representation and dynamics of the neurons are described as follows.

The network is composed of two excitatory neurons connected recurrently. The conduction delays of the connections are randomly generated integer values between 1 ms and 20 ms. The initial synaptic weights are 0.001 pS. Neurons are excited by external and internal currents. \( I \) represent the sum of these two current sources. The dynamics of the neurons are described by the Izhikevich spiking neural network\(^{12} \). \( v \) symbolized the membrane potential and \( u \) represents the membrane recovery. The initial membrane potential was \( v = -65 \text{ mV} \).

\[
\begin{align*}
    v' &= 0.04v^2 + 5v + 140 + u + I \\
    u' &= a(bv-u) \\
    \text{if } v \geq +30 \text{ mV}, \text{then } \begin{cases} v \leftarrow v + c \\ u \leftarrow u + d \end{cases}
\end{align*}
\]

When an action potential reaches an apex of 30 mV, a neuron is assumed to fire. Afterward, the membrane potential is assigned its initial value again and the neuron resets. Four parameters \( a, b, c \) and \( d \) affect the type of the spikes. \( (b, c) = (0.2, -65) \) and \( (a, d) = (0.02, 8) \) are specific values for cortical excitatory neurons with a regular spiking pattern. The parameters and their default values are presented in the following table.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Default Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Excitatory Neurons</td>
<td>( N_{\text{exc}} )</td>
<td>2</td>
</tr>
<tr>
<td>Time Step</td>
<td>( T )</td>
<td>1 ms</td>
</tr>
<tr>
<td>Membrane Time Constant</td>
<td>( \tau_m )</td>
<td>20 ms</td>
</tr>
<tr>
<td>Potentiation time constant</td>
<td>( \tau_+ )</td>
<td>+20 ms</td>
</tr>
<tr>
<td>Depression time constant</td>
<td>( \tau_- )</td>
<td>-20 ms</td>
</tr>
<tr>
<td>Synaptic Time Constant</td>
<td>( \tau_s )</td>
<td>5 ms</td>
</tr>
<tr>
<td>Spiking Threshold</td>
<td>( V_{\text{th}} )</td>
<td>30 mV</td>
</tr>
<tr>
<td>Resting Membrane Potential</td>
<td>( V_r )</td>
<td>-65 mV</td>
</tr>
<tr>
<td>Maximum Synaptic Weight</td>
<td>( w_{\text{max}} )</td>
<td>20 pS</td>
</tr>
<tr>
<td>Minimum Synaptic Weight</td>
<td>( w_{\text{min}} )</td>
<td>1E-3 pS</td>
</tr>
<tr>
<td>Positive Learning Rate</td>
<td>( a_+ )</td>
<td>1E-4</td>
</tr>
<tr>
<td>Negative Learning Rate</td>
<td>( a_- )</td>
<td>1E-4</td>
</tr>
</tbody>
</table>

Table 1. The parameters and their default values.
The learning rules affecting the synaptic weight are STDP and a non-Hebbian learning rule. The arrival of a presynaptic spike affects the postsynaptic membrane positively and induces an EPSP on it, represented by $I_{syn}$. The negative effect of non-Hebbian learning rule on the synaptic weight is considered when the presynaptic spike causes the postsynaptic firing to increase. The arrival of presynaptic spike before the postsynaptic firing results in growing synaptic weight, potentiation, and the reverse order depresses the synaptic weight. The weight changes are represented by $sd$. The synaptic weight is updated regarding each time step using the following equation:

$$s = s + 0.01 + sd$$

$$sd = 0.9sd$$

The constant 0.01 is the activity independent increase of synaptic weight, as used in the Izhikevich model\textsuperscript{12}. The hard bounds imposed on the synapses keep weights between 0.001 and 20 pS. The first MATLAB code box shows the initial values of the parameters related to the structure of the network. The second MATLAB code box shows implementation of the neural dynamics and synaptic learning rules.

```matlab
% Forming three synaptic modifications: Bidirectional, unidirectional and quiescent synaptic weight with the stimulation of sine modulated determinist stimuli. The cell dynamics are based on the Izhikevich spiking neural model and the implemented learning rule is STD.

% Preparation
D=20;               % maximal conduction delay
Ne=2; Ni=0;         % total number
a=0.02*ones(Ne,1);  % excitatory neurons
d=8*ones(Ne,1);     % inhibitory neurons
sm=20;              % maximal synaptic strength
post=ceil([N*rand(Ne,M);Ne*rand(Ni,M)]); % synaptic weights
s_origin=0.001*ones(Ne,1); % synaptic weights
sd=zeros(N,1);      % their derivatives
delays=1+ 19*rand(N,1);
For i=1:Ne
  For j=1:size(delays,2)
    pre{post(i, j)}(end+1) = N*(delays(i, j)-1)+i;
  End;
End;
firings=[-D, 0];    % Spike timing.
a_posi=0.0001; a_nega=0.0001; % Positive and negative learning rates.
tau_pos=20; tau_neg=20; % Potentiation and depression time constant.
to_g=10;            % Synaptic time constant.
v = -65 *ones(N,1); % initial values of Membrane potential
u = 0.2.*v; V_th=30; % initial values
V_r=-65; to_m=20;    % Firing threshold.
St=[6 6]; g_E= 2*(10^-2); % Resting potential.
t_max=300*E+3;      %Membrane time constant.
Amp=[20 20];       %Stimulation time step.
g_E= 2*(10^-2);    %Stimulation amplitude.
The maximum synaptic conductance.
```
For $t=1:t_{\text{max}}$

$I=\text{zeros}(N,1)$; %Frequency of Stimulation.

$\text{Frequance}=[30.1\ 30.1]$;

$s1 = \max((\sin(2\ast\pi\ast\text{Frequance}(1)\ast t/1000)), 0)$;
$s2 = \max((\sin(2\ast\pi\ast\text{Frequance}(2)\ast t/1000)), 0)$;

$\text{If} \mod(t,\text{St}(1))==0 \&\& \mod(t,\text{St}(2))==0$

$I(1,1) = \text{Amp}(1)\ast s1 \ast 8/\pi$;
$I(2,1) = \text{Amp}(2)\ast s2 \ast 8/\pi$;

$\text{End}$

$fired = \text{find}(v>=30)$; %Indices of fired neurons

$v(fired)=-65$;
$u(fired)=u(fired)+d(fired)$;

$\text{firings}=[\text{firings};\text{t}\text{'ones}(\text{length}(\text{firings}),1),\text{fired}]$;

%Firing Pattern

$\text{If} \text{length}(\text{firings})>=1$

$\text{For }r=1: \text{length}(\text{fired})$

$\text{Firing Pattern}(\text{fired}(r),1)=[\text{Firing Pattern}(\text{fired}(r),1),t]$;

$\text{End}$

%Firing Frequency

$\text{For }ii=1:N$

$\text{If} \text{length}(\text{Firing Pattern}(\text{ii}))>=1$

$\text{Freq}=1$;

$\text{ElseIf} \text{length}(\text{Firing Pattern}(\text{ii}))>1$

$\text{Freq}=(\text{Firing Rate}(\text{ii})(\text{end})/\exp(t-\text{Firing Pattern}(\text{ii})(\text{end})/250))+1$;

$\text{End}$

$\text{Firing Rate}(\text{ii})=[\text{Firing Rate}(\text{ii}),\text{Freq}]$;

$\text{End}$

%Non Hebbian Learning Rule

$k=\text{size}(\text{firings},1)$;

$\text{while }k>1$

$\text{If} \text{Firing Rate}(k)(t)>\text{Firing Rate}(k)(t-1)$

$\text{If} t-\text{firings}(k,1)==\text{delays}(\text{firings}(k,2),1)$

$A\_neg=s(\text{firings}(k,2))\ast a\_neg$;

$B=\left(\frac{A\_neg \ast \text{tau}_n \ast \text{to}_s}{\text{to}_m \ast (\text{V}_r-\text{V}_s) \ast (\text{tau}_n \ast \text{to}_s)}\right)$;

$\text{sd}(\text{pre}_fired(\text{firing}(k,2),1))=\text{sd}(\text{pre}_fired(\text{firing}(k,2),1)) - A\_neg \ast \text{exp}(-t/\text{tau}_n)$;

$\text{End}$

$\text{End}$

$k=k-1$;

$\text{End}$

%Hebbian Potentiation

$F=\text{length}(\text{fired})$;

$\text{For }FF=1:F$

$\text{pre}_fired=\text{mod}(\text{pre}_fired(FF)),N)$;

$\text{If} \text{Length}(\text{Firing Pattern}(\text{pre}_fired,1))>=1$

$\text{pre}_arrival=\text{Firing Pattern}(\text{pre}_fired,1)(\text{end})+\text{delays}(\text{pre}_fired)\ast \text{ind}$;

$\text{delta}_t=\text{pre}_arrival-t$;

$\text{If} \text{delta}_t<=0 \&\& \text{delta}_t>=-20$

$A\_pos=s(\text{pre}_fired)\ast a\_pos$;

$\text{sd}(\text{pre}_fired)=\text{sd}(\text{pre}_fired)+(A\_pos \ast \text{exp}(\text{delta}_t/\text{tau}_p))$;

$\text{End}$

$\text{End}$

$\text{End}$

$\text{End}$

%Hebbian Depression

$I\_synapse=\text{zeros}(2,1)$;

$k=\text{size}(\text{firings},1)$;

$\text{While }k>1$

$I\_synapse=[0;0]$;

$\text{If} t-\text{firings}(k,1)==\text{delays}(\text{firings}(k,2),1)$

$I\_synapse=g\_syn \ast (v(\text{post}(\text{neuron},1))-E)$;

$\text{End}$

$\text{If} \text{length}(\text{Firing Pattern}(\text{post}(\text{firings}(k,2)),1))>1$

$\text{delta}_t=\text{pre}_arrival-\text{Firing Pattern}(\text{post}(\text{firings}(k,2)),1)(\text{end})$;

$\text{If} \text{delta}_t>0 \&\& \text{delta}_t<20\ast \text{ind}$

$A\_neg=s(\text{neuron})\ast a\_neg$;

$\text{sd}(\text{firings}(k,2))=\text{sd}(\text{firings}(k,2)) - \text{abs}(A\_neg \ast \text{exp}(-(\text{delta}_t/\text{tau}_n))$;

$\text{End}$

$\text{End}$

$k=k-1$;

$\text{End}$

$\text{End}$

$\text{End}$

$v=v + ((0.04\ast v+5)\ast v+140-\text{u}+I)$;
$u=u + a\ast(0.2\ast v-u)$;
$s=\max(0.001,\text{min}(s\_\text{origin}+\text{sd}))$;
$s\_\text{origin} = s$;
$\text{sd} = 0.9 \ast \text{sd}$;

$\text{End}$
References


