Development of a Tangible Human-Machine Interface
Exploiting In-Solid Vibrational Signals Acquired by Multiple Sensors

M A R C O  F A B I A N I

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Development of a Tangible Human-Machine Interface Exploiting In-Solid Vibrational Signals Acquired by Multiple Sensors

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Abstract

We all use tangible controls (that means controlled by our hands) every day to operate electronic devices. Typical examples of this sort are keyboards, mice, touch pads and touch screens. All such interaction devices share the disadvantage of being equipped with mechanical or electronic devices exactly at the point of interaction with the interface (switches, potentiometers, sensitive layers, force-resistive sensors, etc). This tends to increase the manufacturing costs, limit the interaction to predetermined regions of the contact surface, and make them less sturdy and less suitable for outdoor/industrial operation.

The aim of this project (that contributes to the Tai-Chi project) is to find a novel method to approach the development of tangible interfaces that reduces the need of intrusive sensors/devices. To do this, we directly acquire and study the vibrations produced by the interaction of the user and that propagate inside the material.

The techniques that are commonly used do not take into account any apriori knowledge about the vibrations, and for this reason they are usually not precise and they are not able to track a continuous interaction.

By taking into account the dispersive properties of the vibrations propagating inside solids (and actually measuring the dispersion curves), we developed a robust method that exploits the dispersion of the vibrations in order to localize an impulsive interaction (finger tap) and to track a continuous one (finger scratch) on a thin plate of an almost isotropic material. We then used this method to develop a tangible interface by applying four accelerometers on a MDF plate.
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Chapter 1

Introduction

We all use tangible controls every day to operate electronic devices of all sorts or to control electrical appliances. In fact, we access interactive kiosks, we use personal digital assistants, we dialog and interact with automatic tellers, etc. The vast majority of input interfaces are tangible, as they are controlled by our hands. Typical examples of this sort are keyboards, mouses, touch pads and touch screens. All such interaction devices share the disadvantage of being equipped with mechanical or electronic devices exactly at the point of interaction with the interface (switches, potentiometers, sensitive layers, force-resistive sensors, etc). This tends to increase the manufacturing costs, limit the interaction to predetermined regions of the contact surface, and make them less sturdy and less suitable for outdoor/industrial operation.

The Tai-Chi project (Tangible Acoustic Interfaces for Computer-Human Interaction) aims at finding novel methods and solutions that reduce the need of intrusive sensors/devices in the area of contact and interaction ([2]). This thesis is a contribution to that Project.

In the early stage of the Tai-Chi research, the attention was focused on the localization of single, impulsive interactions, e.g. finger tapping on the surface of a panel, through the study of the vibrations that propagate inside the material. The techniques that are commonly used to achieve this purpose, like for example TDOA (Time Delay Of Arrival, see section 5.1), are based on the detection of the first arrival of the wavefronts to an array of sensors. From the time differences, with geometrical or mathematical methods, it is possible
to estimate the position of an impulsive tactile interaction (see section 5.3).

Anyway, there are a few problems related to the use of these classical techniques:

- The lack of precision, caused by the phenomenon of dispersion (different frequencies propagate at different speeds) that is not taken into account.
- The elastic propagation inside thin plates, that are the objects used as interfaces, is not deeply studied and not used for the estimation of the tactile interaction.
- Only a small part of the signal (e.g. the wavefront) is used to estimate the position of the interaction.
- In a complete tangible interface, another task has to be accomplished: the tracking of a continuous source induced, for example, by an object scratching the surface. The classical methods are not employable to solve this problem.

To better understand the types of signals produced by the various interactions, and to exploit their properties, we first study the theory of wave propagation in thin plates (see chapter 3). Thin plates can be considered as elastic waveguides, where modal waves are propagating. In particular, in the range of frequencies concerning the vibrations produced by tactile interactions, we can isolate a single mode. This wave mode is dispersive, and for this reason we also deepen the study of the dispersion (section 3.4), setting up an experiment (chapter 4) to validate the theory with the observations.

We focus our attention on a few specimens and we measure experimental curves in order to extract the elastic properties of the materials to validate the results with the theoretical values. This information is necessary to model the impulse response of the thin plates.

We then propose a novel method (RDOR, see chapter 5) that uses the dispersion to estimate the position of the interaction. RDOR (Relative Distance Of Receivers) exploits the fact that dispersion depends on the traveled distance of the signals. We define relative distance between two sensors the difference between their distances from the source. The signal acquired by
the receiver that is farther from the source presents an additional dispersion if compared to the one acquired by the closer receiver. This additional dispersion can be compensated by a filtering operation. In practice, we can create a bank of filters in which to each filter corresponds a relative distance between the considered sensors. The best estimate of this distance is obtained by searching the optimal compensation filter. These operations are then repeated for each pair of available receivers, obtaining an estimate of their relative distances. We can finally estimate the position of the source from the relative distances using Tarantola’s approach to formulate the estimation problem as an inversion (see section 5.3). The main advantages of this novel method over the classical ones are:

- It is more precise.
- It takes into account the theory of wave propagation in thin plates.
- It exploits the entire signal.
- It can be applied to the tracking problem because it does not rely on the wavefronts.

We solve the tracking task by using iteratively the RDOR method on the acquired continuous signal, divided into several segments, corresponding to different time windows (see section 5.2). The actual source position is determined starting from the estimate position at the previous step. We also propose a few optimizations that make the use of the RDOR method feasible for real time applications (the basic approach is very resource-demanding, see section 5.4). By employing the RDOR technique it is possible to develop a complete Human - Machine tangible interface, able to detect both types of interactions: impulsive and continuous (see section 5.5).

We tested the RDOR technique with proper experiments on different specimens (see chapter 6). The results prove that the method is stable and precise: the error of estimation is always very low.

We can conclude that the approach here proposed solves the main problems related to the classical detection methods. We suggest further developments in order to obtain a reliable Tai-Chi interface, that produces accurate and efficient results (see chapter 7).
2.1 A brief overview on Tangible Interfaces

The underlying aim of this project is to create an alternative Human - Machine Tangible interface that does not rely on mechanical and/or electronic devices at the interaction point with the interface (e.g. switches, potentiometers, sensitive layers, force resistive sensors, etc.). There are several reasons for this: the first problem with nowadays interfaces is the cost, that makes their extension to large dimensions not convenient. A second reason to avoid classic construction approaches is the limited usability in critical environments (public areas, outdoor work places, kitchen, etc.), where, for example, heavy duty usage or hygienic conditions are a major thread (it can be dangerous to use a normal keyboard, known to be very anti-hygienic, in medical environments). Many types of interfaces are in commerce nowadays, each one intended for a more or less specific application. Some of them have been on the market for a long time. They might be considered the state of the art in this field and can be taken as an example in terms of functionalities (beside the standard keyboards and mouses): these are touchscreens and graphic tablets. A brief description of other devices can be read in [27].

**Touchscreen ([1])** Touchscreen monitors have become more and more common in everyday life as their price has steadily dropped in the last years. Nonetheless, these devices are still rather expensive (because of the technology
that is used) and of limited dimensions. There are basically three techniques to recognize a person’s touch:

- **Resistive**: consists of a normal glass panel that is covered with a conductive and a resistive metallic layer. These two layers are held apart by spacers, and a scratch-resistant layer is placed on top of the whole setup. An electrical current runs through the two layers while the monitor is operational. When a user touches the screen, the two layers make contact in that exact spot (Fig. 2.1). The change in the electrical field is noted and the coordinates of the point of contact are calculated by the computer.

- **Capacitive**: a layer that stores electrical charge is placed on the glass panel of the monitor (Fig. 2.2). When a user touches the monitor with her finger, some of the charge is transferred to the user, so the charge on the capacitive layer decreases. This decrease is measured in circuits located at each corner of the monitor. The computer calculates, from the relative differences in charge at each corner, exactly where the touch event took place.

- **Surface Acoustic Waves**: two transducers (one receiving and one sending) are placed along the x and y axes of the monitor’s glass plate (Fig. 2.3). Also placed on the glass are reflectors, they reflect an electrical signal sent from one transducer to the other. The receiving transducer is able to tell if the wave has been disturbed by a touch event at any instant, and can locate it accordingly.

It is important to note that all the described techniques can be considered *active*: there is a constant current or vibration passing through the material and it is possible to detect the position of the interaction only when one of these is perturbated. The presence of all the devices used to produce and analyze these perturbations keeps the price of the touchscreens high.

**Graphic Tablet ([3])** A graphic tablet (or digitizing tablet) is a computer peripheral device that, originally, was meant to allow one to hand-draw images directly into a computer, generally through an imaging program. Today,
Figure 2.1: Functioning principle of a resistive touchscreen (from [5]).

Figure 2.2: Functioning principle of a capacitive touchscreen (from [5]).

graphic tablet are used also as a replacement for mouses. Graphics tablets consist of a flat surface upon which the user may "draw" an image using an attached stylus, a pen-like drawing apparatus. The first graphic tablet resembling contemporary tablets was the RAND Tablet: it employed a grid of wires under the surface of the pad that encoded horizontal and vertical coordinates in a small magnetic signal. The stylus would receive the magnetic signal, which could then be decoded back as coordinate information. Modern graphic tablets operate in a fashion similar to the RAND Tablet. In modern
devices, though, the horizontal and vertical wires of the grid are separated by a thin insulator. When a pressure is applied to the tablet, the horizontal wire and vertical wire associated with the corresponding grid point meet each other, causing an electric current to flow into each of these wires. Since an electric current is only present in the two wires that meet, a unique coordinate for the stylus can be retrieved. Graphic tablets, as touchscreens, are active devices, thus rather expensive.

Beside the problem of costs (and dimensions), active devices suffer from the intrinsic fragility of the several electronic devices used to build them. For these reasons, a new kind of interface should have the following characteristics:

- To be scalable in dimensions in a cheap and effective way.
- To be built with materials and devices that allow it to be suitable for any condition and environment.
- To reduce the costs, relying on a passive technique (e.g. to use directly the perturbation caused by the interaction between the user and the material).

These points led to some new approaches proposed during the Tai-Chi project’s development (see [27]):

- LTM (Location Template Matching): the point of interaction is found by comparing the signal with a series of known templates associated with
different positions and acquired in a calibration phase run previously.

- **TDOA (Time Delay of Arrival):** the time difference of arrival of the vibration to an array of sensors is estimated with a threshold’s crossing method. An algorithm calculates the interaction position from these time differences and the speed of the wave.

- **Other approaches:** Ultrasonic tactile sensor, Fingertip Gesture Interface, PingPongPlus, Acoustic Imaging, Surface Acoustic Wave, hybrid techniques TDOA/LTM, . . .

These techniques are passive as opposite to those used in touchscreens and tablets, but not all of them are also cost- and time effective.

### 2.2 Contact sensors

A passive technique implies the detection and acquisition of the vibration induced by the user’s interaction and propagated inside the material. Contact sensors (or transducers) are applied to the surface of the interface and employed for this task. The following characteristics allow us to do a classification of the sensors:

- **Measured variable** (e.g. displacement, velocity, acceleration).

- **Construction** (e.g. used materials): the most common sensors are built using piezoelectric crystals (for example quartz). These materials, when mechanically deformed, produce a voltage difference between their ends that is proportional to the amount of deformation. There are also other types of piezoelectric materials, like ceramics and microfilms, that are usually artificial and act in the same way.

Another kind of sensor is build using MEMS technology (Micro-Electro-Mechanical Systems, [9]). MEMS is the integration of mechanical elements, sensors, actuators, and electronics on a common silicon substrate through micro-fabrication technology.

Other types of sensors are contact microphones (electromagnetic microphones, condenser microphones etc.), like the ones used to amplify
acoustic musical instruments. These are classic microphones purposely modified to be applied on the surface of an object.

• How the variable is measured. Usually the value of the variable is measured from the direct effect of the vibration on the sensor. For example, if a piezoelectric crystal is used, the output from the sensor is the electric tension produced by the crystal itself (properly conditioned) when deformed by a vibration. Another way to approach the measurement is to employ two very close sensors. The first sensor emits an ultrasonic signal that is received by the second one. The ultrasonic signal is modified by the interaction with the vibration that has to be measured. Its value is the difference between transmitted and received signals. This type of measurement is used for example in TSM or SH-SAW sensors ([10]).

We think that two types of sensors are suitable for the applications studied during this work:

• **Accelerometers**: the accelerometers measure the acceleration of the particles at the application point. Accelerometers are widely used in those fields that require measurements of vibrations inside solid materials, especially for NDE (non-destructive evaluation, for example investigation of micro-cracks in metal or concrete parts, study of vibrations in machinery parts, cars, air crafts etc.). Accelerometers come in a great variety of types: there are models measuring acceleration on 1, 2, or 3 axis/directions, accelerometers with very tough shells capable to resist in very stressful conditions, of different sizes and shapes. Moreover, from acceleration, with classic cinematic laws, both velocity and displacement can be derived with one and two integrations, respectively. Accelerometers are usually built with piezoelectric materials.

The accelerometer Knowles Electronics BU-1771 has been chosen for this research (in figure 2.4 the sensor attached to an MDF panel). This is a piezo ceramic accelerometer with an integrated operational amplifier that guarantees a good output level at the cost of a power supply. Inside the shell, a bridge is connected to an inertial mass, to which are
attached two thin piezo ceramic stripes with opposite polarities, capable to measure both positive and negative accelerations. When a vibration reaches the sensor, the bridge bends, and thereby the piezoelectric material deforms, while the inertial mass remains substantially motionless. The piezoelectric stripes produce a voltage difference that is amplified and sent to the output.

The sensitivity of this accelerometer is acceptable (−45 dB referred to 1 Volt/g) and the frequency response is flat up to 3 kHz, while the span of frequencies that can be used is up to 8 ± 9 kHz (the resonance peak
is at $11 \div 12 \ kHz$, see figure 2.5).

The steel shell makes this accelerometer tough and its rectangular shape simplifies the application. A drawback of this sensor is its price, but for testing purposes, where good precision is needed, this accelerometer proved to be very suitable. For more detailed characteristics see [13] and [14].

- **Piezoelectric diaphragms**: an alternative and cheap choice for sensors might be the piezoelectric diaphragms (see Fig. 2.6), that are normally used as actuators (for example internal speaker of PCs) but can be used also as receivers. We performed some measurements of material’s properties (see section 4.1) using a piezoelectric diaphragm to transmit a well defined vibration to the interface. Diaphragms are built with a very thin layer of piezoelectric material (white part in figure) mounted on a metallic base. Because of the direct piezoelectric effect (piezo-generator), when the layer is deformed, a voltage difference is produced at its ends. On the other hand, because of the inverse piezoelectric effect (piezo-motor), a tension at the two ends of the crystal produces a vibration/deformation of it. A problem arising with the use of these sensors is the very low signal level obtained at their outputs: for commercial applications, anyway, this can be solved during the design process.
Figure 2.6: Various piezoelectric diaphragms.
Chapter 3

Wave propagation in solids

A vibration propagating inside a solid is generally called an *elastic wave* (sometimes also called *acoustic wave*, although this is partly wrong because only some types of elastic waves are acoustic). Our attention is focused only on the propagation in solid materials because the purpose of this research is to develop a tangible interface. Unlike for gases (e.g. air), elastic waves in solids are more complex for a series of reasons. First of all, the propagation depends on the geometry of the object: the study of wave propagation usually starts by assuming objects of infinite dimensions, but this assumption is not valid for real situations, and in particular for solids, that are normally of limited dimensions. Moreover, in solids several wave modes propagate depending on the geometry of the object and the material’s properties. Finally, some of these modes suffer the phenomenon of *dispersion* (signals at different frequencies propagate at different speeds).

3.1 Wave Equation

In general terms, a wave is a physical perturbation (in the case of elastic waves a movement of particles) that propagates through the medium, producing a subsequent effect on near particles. Any type of wave follows the same law that can be synthetized by the *wave equation*.

An oscillatory phenomena can be represented as a generic continuous
curve:

$$\xi = f(x)$$  \hspace{1cm} (3.1)

By introducing a time variable, its evolution in space (along $x$ axis) and time ($t$) can be represented by:

$$\xi(x, t) = f(x \pm vt)$$  \hspace{1cm} (3.2)

where $v$ is called *wave speed* and the "±" corresponds to the two possible directions of propagation.

If $\xi(x, t)$ is a sinusoidal or harmonic function, the 3.2 can be written as:

$$\xi(x, t) = \xi_0 \sin k(x - vt) = \xi_0 \sin([k(x - vt) + 2\pi])$$  \hspace{1cm} (3.3)

where $\xi_0$ is the amplitude of the sinusoid and $k$ is called the *wave number*, from which the wave length $\lambda$ can be obtain by:

$$\lambda = \frac{2\pi}{k}$$  \hspace{1cm} (3.4)

The wave length represents the distance after which the perturbation is repeated.

The 3.3 can be also represented in terms of pulsation $\omega$, that is related to $\lambda$, $k$ and frequency $f$ by:

$$\omega = kv = \frac{2\pi v}{\lambda} = 2\pi f$$  \hspace{1cm} (3.5)

The perturbation equation becomes:

$$\xi(x, t) = \xi_0 \sin(kx - \omega t)$$  \hspace{1cm} (3.6)

Only periodic functions has been taken into account. The Fourier Theorem can help us to extend the theory to non period functions: any ondulatory motion can be expressed by a linear superposition of an infinite number of harmonic waves at frequencies multiples of a fundamental frequency. In these terms, a non periodic signal (like a tap or a scratch on a table) can be seen as a periodic signal with an infinite period.
The *differential wave equation*, that defines the wave at any point in space and time, can be written as:

\[
\frac{\partial^2 \xi}{\partial t^2} = v^2 \frac{\partial^2 \xi}{\partial x^2} \tag{3.7}
\]

The solution to equation 3.7 is known to be:

\[
\xi(x,t) = f_1(x - vt) + f_2(x + vt) \tag{3.8}
\]

Equation 3.8 shows the general case when two signals propagate in opposite directions. If the signal propagates in only one direction, either \(f_1\) or \(f_2\) can be used.

The vectorial form:

\[
\frac{\partial^2 \xi}{\partial t^2} = v^2 \nabla^2 \xi \tag{3.9}
\]

is another common way to write the 3.7.

Using these concepts, we can derive the equations for elastic waves.

### 3.2 Elastic waves

The propagation of a vibration inside a medium is possible because the movement of one particle forces the particles around it to move as well. The particles constituting solids are connected to each other with forces that are stronger than in liquids and gases. These forces (or links) between particles can be seen as rubber bands or springs: the structure is then partly movable and the stress and strains on the links produced by a vibration can propagate from one particle to the next one. To express these concepts in mathematical terms we can use two fundamental laws: the well known inertial principle and the Hooke’s law (elasticity law) in the generalized form (anisotropic form). Hooke’s law states that “the power of any springy body is in the same proportion with the extension” ([7]). Cauchy generalized Hooke’s law to three dimensional elastic bodies and stated that the 6 components of stress are linearly related to the 6 components of strain. By considering an infinitesimal cube of material, its displacement can be calculated using inertial principle and Hooke’s law. The theoretical treatment of this part is beyond the scopes
of this work, so only the final result is presented. It is important to know that applying Hooke’s law, it is possible to express the stress and strains of the infinitesimal cube with a tensor, know as elastic tensor. If we consider the symmetries of the problem and assume for simplicity to have an isotropic material (the waves are propagating in the same way in every direction, generating a spherical wave front), the elastic tensor will have only two independent components that can be expressed using the Lamé parameters, $\lambda$ and $\mu$. We can then write the elastic waves equation for solids as (see [19] and [16]):

\[
\frac{\lambda + 2\mu}{\rho} \nabla^2 \theta + \frac{\mu}{\rho} \nabla^2 \nabla u = \frac{\partial^2 u}{\partial t^2} \tag{3.10}
\]

The variable $\theta$ represents the divergence of displacement $u$ (displacement is a vector because it is considered in 3 dimensions), that is $\theta = \nabla u$. $\rho$ is the density of the material and $\lambda$ and $\mu$ are the Lamé parameters, that can be derived from two elastic properties of the material, that are Young’s modulus ($E$) and Poisson coefficient ($\nu$), by ([4]):

\[
\lambda = \frac{\nu E}{(1 + \nu)(1 - 2\nu)} \tag{3.11}
\]

\[
\mu = \frac{E}{2(1 + \nu)} \tag{3.12}
\]

Young’s modulus $E$ is defined as the ratio between strain and stress, while Poisson coefficient is dependent, among other relations, by the ratio between the speed of propagation of two wave modes, $P$ and $S$.

In fact, elastic waves differentiate from vibrations propagating in gases because several modes of propagation are present. A classification, based on these modes, can be done for objects that are assumed to be semi-infinite solid half-spaces (Fig. 3.1, from [17]):

- **Bulk Waves**: these waves propagate deep inside the material. They can be further divided into:
  - Longitudinal (P waves or compression waves): particles are moving in the direction of propagation. This type of wave is very similar to compression waves propagating in gases (e.g. sound) (Fig. 3.4).
Figure 3.1: Propagation of different elastic waves modes inside a solid object (semi-infinite solid half-space).

Figure 3.2: Particle movement caused by the P waves.

The speed of P waves can be derived from Lamé's parameters and density with the following relation:

\[ v_\alpha = \sqrt{\frac{\lambda + 2\mu}{\rho}} \]  

(3.13)

The longitudinal bulk elastic waves can be also called acoustic waves.
Figure 3.3: Particle movement caused by the S waves.

- Transverse (S waves or shear waves): particles are moving perpendicular to the direction of propagation (Fig. 3.4). The speed of S waves can be obtained with the relation:

\[ v_\beta = \sqrt{\frac{\mu}{\rho}} \]  

(3.14)

It has to be noted that \( v_\beta < v_\alpha \). Both P and S wave modes are non-dispersive.

- **Surface Waves**: these waves propagate only close to the surface of the object as a circular wave from the point of contact (the waves produced by a stone thrown in the water are an example of surface waves). They derive from the recombination of Bulk Waves near the surface. Surface Waves can be also divided into different modes:

  - Rayleigh waves: these waves travel the surface of a relatively thick plate of solid material penetrating to a depth of about one wavelength (\( \lambda \)). The particles movement has an elliptical orbit (Fig. 3.5 and Fig. 3.6) and horizontal motion. Rayleigh waves are a combination of P and SH waves, where SH and SV are the horizontal and vertical components of S waves.

Let the plate have thickness 2a. Then there are a certain number of permitted frequencies that can propagate and solve the transcen-
dental equation:

\[(k^2 + \beta^2)^2 \cosh(\alpha a) \sinh(\beta a) - 4k^2 \alpha \beta \cosh(\beta a) \sinh(\alpha a) = 0\] (3.15)

where \(k\) is the wavenumber, \(\alpha\) and \(\beta\) are defined as:

\[
\alpha \equiv \sqrt{k^2 - \frac{\omega^2}{v_\alpha^2}} \tag{3.16}
\]

\[
\beta \equiv \sqrt{k^2 - \frac{\omega^2}{v_\beta^2}} \tag{3.17}
\]

with \(\omega\) the angular frequency, \(v_\alpha\) and \(v_\beta\) the velocities of P and S modes, \(\rho\) the density of the material, and \(\lambda\) and \(\mu\) the Lamé parameters. In case the solution to 3.15 is imaginary in \(k\), the corresponding real solution for \(k\) can be found from the transformed
Figure 3.5: Particles movement caused by the Rayleigh waves.

Figure 3.6: Rayleigh waves: elliptical movement of the particles.

equation:

\[
\left(2k^2 - \frac{\omega^2}{c^2}\right)^2 \cos(\alpha' a) \sin(\beta' a) + 4k^2 \alpha' \beta' \cos(\beta' a) \sin(\alpha' a) = 0
\]

(3.18)

where

\[
\alpha' \equiv \sqrt{\frac{\omega^2}{v_\alpha^2} - k^2}
\]

(3.19)

\[
\beta' \equiv \sqrt{\frac{\omega^2}{v_\beta^2} - k^2}
\]

(3.20)

(from [4], see also appendix A)

Rayleigh waves are dispersive.
– Love waves: they are a type of surface wave formed by the constructive interference of multiple reflections of SH waves at the free surface. The particle motion for Love waves is parallel to the surface but perpendicular to the direction of propagation. Love waves are dispersive.

– Lateral (or head waves): these are refracted waves. They generate at the discontinuity surface separating a material with low propagation velocity and one with higher velocity (e.g. solid material and air) when the critical angle is reached. The refracted wave does not propagate inside the second material, but inside the thin layer that separates the two materials.

Surface waves are slower than bulk waves, but they have larger amplitude: for this reason they are easier to detect.

These are the principal modes propagating inside solid materials: some other are present as well, but they will not be described here.

All these components, as can be seen in Fig. 3.1, can be very well distinguished in the semi-infinite half space case because there are no reflections from edges mixing with direct waves. When the studied object becomes an infinite plate, thus introducing a second surface (discontinuity) close to the first one, the vibrations reflect on the opposite surface and they make the
distinction of the various modes more complex (see Fig. 3.7). But when the plate is sufficiently thin, the propagation of elastic waves undergoes a series of simplifications.

The system developed during this project is meant to work on objects that can be considered thin plates: imagine to use a shop window, or the surface of a table or another piece of furniture as an interface by applying a few sensors on it. For this reason we need a deep understanding of the wave propagation inside this type of structures.

We can note that further complications in the wave propagation are introduced by reflections from the borders, if we consider the plate also limited in length and width.

3.3 Wave propagation in thin plates

Inside thin plates, only few modes of elastic waves can propagate. This fact leads to a more simple representation of waves inside the object. The most important modes are Rayleigh waves, that when are propagating inside thin plates are commonly known as Lamb waves (or guided waves).

A plate can be defined thin when its thickness is smaller than a certain value. This value, that is proportional to the wavelength $\lambda$ of the waves propagating inside the object, is not very well defined in literature. Some sources suggest to have a wavelength 10 times larger than thickness ($\lambda > 10 \cdot 2a$), some other suggest less stringent limitations (see [25]). Anyway, since $\lambda$ is inversely proportional to frequency $f$ and directly proportional to the wave speed ($\lambda = f/v$), the thin plate condition varies not only in function of the material but also in function of the type of signal that has to be investigated. The larger the frequency is, the thinner the plate has to be, in order to be considered thin. The thin plate’s condition is also limited by the width $w$ and length $l$ of the plate, that must be much larger than $\lambda$ ($w,l >> \lambda$).

Thin plates wave propagation is a very particular case that has to be treated as a standalone argument. This has been done by I.A. Viktorov in [26]. In this chapter we only describe the most useful parts of Viktorov’s theory, while a larger summary can be found in appendix A.
Lamb waves are defined as elastic perturbations propagating inside a solid plate (or layer) with free boundaries, for which the displacement occurs both in the direction of propagation and perpendicularly to the plane of the plate. Lamb waves are one of the types of normal or plate modes propagating inside an elastic wave guide (in this case, a plate), and for this reason they are normally, but mistakenly, called normal modes, despite the fact that other normal modes exists (e.g. transverse normal modes, wherein the motion is perpendicular to the direction of propagation and parallel to the boundaries of the plate). By considering a thin plate as an elastic wave guide, it is possible to imagine Lamb waves as two dimensional representation of a bulk wave bounded by the upper and lower surfaces of the plate.

Lamb waves can be distinguished into two types: extensional (or symmetric, s) and flexural (or antisymmetric, a), each one having an infinite number of modes itself \((s_0, s_1, s_2, \ldots, s_n; a_0, a_1, a_2, \ldots, a_n)\). A representation of Lamb modes can be see in Fig. 3.8.

Usually both modes should be distinguishable inside the same signal, but as observed by Tucker in [25], below a certain frequency only flexural mode propagates, while extensional, that normally has also a lower amplitude compared to a mode, seems to be missing. This phenomenon can be explained if we consider that the employed sensors measure the accelerations in the \(z\) direction. For the longitudinal mode, the component of the displacement in...
the $x$ direction is much larger than the one in the $z$ direction, and for this reason the latter is not detected by the accelerometers (see appendix A.4).

### 3.4 Elastic waves properties

When propagating, elastic waves experience two phenomena that deteriorate the signal, that otherwise would infinitely travel with no modifications. These phenomena are attenuation and dispersion.

**Attenuation**

Attenuation is defined as the decrease of intensity of a signal or wave as a result of absorption of energy by the material and of scattering out of path to the detector (interferences from reflected waves), but not including the reduction due to geometric spreading ([8]). Attenuation is usually defined in terms of decibels lost per unitary length (e.g. dB/m) and is expressed by the *attenuation coefficient* $\alpha$. Attenuation coefficient is a property of the material, and can be measured easily. For a plane wave, attenuation can be expressed as:

$$E_{\text{out}} = E_{\text{in}} e^{-\alpha d} \quad (3.21)$$

where $d$ is the distance traveled by the wave, $E_{\text{in}}$ and $E_{\text{out}}$ are the amplitude of the field respectively before and after traveling distance $d$. It is important to note that usually attenuation is frequency dependent: this leads to an *attenuation curve* rather than a single value for $\alpha$. Since attenuation depends also on the absorption of energy by the material’s particles, solids will have an higher attenuation than liquids and gasses, because of stronger links among them.

**Dispersion**

Dispersion is defined (see [8]) as any phenomenon in which the velocity of propagation of a wave is wavelength dependent. The effect of dispersion on multitone signals is substantially the modification of signal’s spectrum, in particular its phase: if a signal is measured at two different points situated along the same direction of propagation, the signal recorded at the second
sensor will be not only a delayed version of the signal at the first sensor (eventually attenuated), but also a completely different signal. Dispersion depends on the material’s properties and also on the mode of propagation: for example, plate waves are dispersive while bulk waves are not.

To understand dispersion better, we have to clarify a few concepts about wave speeds. In common terminology, three speed definitions are used: wave speed, phase velocity and group velocity. Wave speed is defined as the time spent by the wave to travel from one point to another, divided by the traveled distance. To measure this time, two sensors can be used, and the transit time can be defined by the crossing of an amplitude threshold (see Fig. 3.9). This task is particularly hard in presence of dispersion, because dispersion tends to spread the wave front, thus largely limiting the accuracy of threshold system (this is also a major drawback of localization techniques, like TDOA, see section 2.1).

Phase and group velocities, instead, pertain to the phase point transit time of individual peaks (frequencies) within a signal and the centroid transit time of the signal, respectively (Fig. 3.9). To properly measure these velocities, the separation between the two sensors must be changed by a known distance increment. Phase velocity may then be calculated by dividing the change in sensors separation distance by the change in phase point transit time. The group velocity instead may be calculated by dividing the change in sensor separation distance by the change in centroid transit time. For non-dispersive wave propagation phase velocity and group velocity are equal. For dispersive wave propagation, phase velocity could be either higher or lower than group velocity. A dispersion curve can be defined by calculating phase velocity for each frequency component. Differences in phase and group velocity may be observed by examining a wave packet similar to that in Fig. 3.9. For dispersive wave propagation, the individual peaks within the wave packet will move relative to the centroid as the wave propagates through a medium. Group velocities can be more difficult to calculate because of centroid transit time discrepancies. The centroid location is defined like the energy center of the wave packet. Interference from edge’s reflections with the wave packet can lead to an incorrect calculation of the wave packet centroid. In addition, edge reflections (observed after the enclosed portion of the signal in Fig. 3.9) can
also create difficulties in defining the trailing edge of the wave packet. Phase velocities calculations are generally more accurate because phase points are easier to be located.

Dispersion curves (Fig. 3.10) are commonly presented by plotting phase velocity on the $y$ axis and the frequency-thickness ($fh$) product on the $x$ axis. Higher modes possess distinct cutoff frequencies below which propagation does not occur; therefore, the fundamental modes may be isolated by using low excitation frequencies (and/or thin plates). Fig. 3.10 illustrates that the flexural mode phase velocity is greatly affected by changes in the frequency-thickness product, unlike the extensional mode.

Dispersion relations describing how velocity varies with frequency and thickness may be derived using a fundamental elasticity approach that enforces boundary conditions on the top and bottom surfaces of the plate. The elasticity approach results in exact dispersion relations and the relations are derived in terms of bulk wave speeds.

To obtain dispersion curves for Lamb waves, the solution to the following characteristic equation has to be found, in this case in a numerical way. Considering $2a$ as the thickness of the plate, $k_\alpha$ and $k_\beta$ respectively as the wave
numbers of bulk modes P and S, \(v_s\) and \(v_a\) as the phase velocity of Lamb waves (\(s\) and \(a\) modes), we have for Lamb symmetrical modes (see appendix A):

\[
\frac{\tan(\bar{d})\sqrt{(1 - \xi^2_s)}}{\tan(d)\sqrt{\xi^2_s - \xi^2_s}} + \frac{4\xi^2_s\sqrt{(1 - \xi^2_s)\sqrt{\xi^2_s - \xi^2_s}}}{(2\xi^2_s - 1)^2} = 0
\]

(3.22)

and for antisymmetrical modes:

\[
\frac{\tan(\bar{d})\sqrt{(1 - \xi^2_a)}}{\tan(d)\sqrt{\xi^2_a - \xi^2_a}} + \frac{4\xi^2_a\sqrt{(1 - \xi^2_a)\sqrt{\xi^2_a - \xi^2_a}}}{(2\xi^2_a - 1)^2} = 0
\]

(3.23)

where:

\[
\bar{d} = k_{\beta}d
\]

(3.24)

\[
\xi^2_{s,a} = \frac{v_{s,a}^2}{v^2_{s,a}}
\]

(3.25)

\[
\xi^2 = \frac{v_{\beta}^2}{v^2_{\alpha}}
\]

(3.26)

Many authors performed calculations of the phase velocities and their dependence on the plate thickness and frequency (dispersion curves); to achieve this purpose the elastic properties of the medium (in this case the velocities \(v_{\alpha}\) and \(v_{\beta}\) of the P-wave and S-wave in the boards) are necessary. Dispersion curves for the specimens used in this thesis project are measured and calculated as
explained in chapter 4.

3.5 Materials

The methods to build tangible interfaces that are presented in this work aim to work on common objects, for example a kitchen table, a window, or the surface of any object that can be found on the working place. These objects are usually made of thin plates of wood (massive and composite), metal, plastic and glass. To choose some specimens to be studied we make a few considerations.

Massive wood is a common material, but it is highly anisotropic: for this reason, the methods based on the time delay of arrival or similar, like the ones employed during this work, cannot be used, because the waves propagating in different directions are traveling at different speeds and are not following straight paths. Other techniques can be employed with anisotropic materials, like for example LTM (see section 2.1).

Anyway, other materials containing wood are very widely used to build furnitures and other objects. They are composite materials. Although they might appear also anisotropic, some of them, like for example MDF (Medium Density Fiberboard), can be assumed to behave in an isotropic way, because of the very small size of the particles composing them. For this reason, the main object of investigation during this work is MDF, a type of hardboard which is made from wood fibers glued under heat and pressure. MDF has various advantages: it is dense, flat, stiff, it has no knots and it is easily machined. Because it is made up of fine particles it does not have an easily recognizable surface grain and it can be cut, drilled, machined and filed without damaging the surface. It may be dowelled together and traditional woodwork joints may even be cut and glued together with PVA wood glue. MDF can be painted to produce a smooth quality surface: oil, water-based paints and varnishes may be used on it. Veneers and laminates may also be used to finish it ([6]). Moreover, MDF has a very high attenuation coefficient, that is useful to prevent disturbance from edge’s reflections (see section 3.4) and waves are propagating at a relatively slow speed, making them easily measurable.

We performed also a few test on plexiglass, another isotropic material well suitable for these applications. We could have used glass, but we preferred
plexiglass because the waves are propagating with lower speed, it is less fragile and can be more easily cut. Another advantage of plexiglass is its transparency that allows for example the back projection of images on its surface. Plexiglass has lower attenuation if compared with MDF so the edge’s reflections have an higher impact on the detected signals. Another disadvantage is that a continuous interaction, like the ones studied in this project (e.g. scratching the surface with an object or with finger), on a smooth surface like plexiglass, has a very low energy level and it is very hard to distinguish from the background noise of the transducers. On the other hand, MDF has a partially rouge surface, and the excitation is well distinguishable. The use of a plexiglass panel with a rough surface could be an alternative (polished surface). Two types of plexiglass have been tested, a thicker (6 mm) transparent one and a thinner (2 mm), white one (a particular of the two plexiglass plates can be seen in Fig. 3.12).

The system, anyway, is independent of the material as long as it is isotropic and its properties (Young’s modulus and Poisson’s ratio, see section 3.4) are known. For this reason, the application of the methods developed here is possible also on other materials, like metal or glass, as long as an impulse or a continuous source is detectable.
Figure 3.12: Particular of the two plexiglass plates (white one is thin, transparent is thick) used during the experiments, over the foam rubber.
Chapter 4

Study of the dispersion

Dispersion is a phenomenon that compromises the precision of estimation when classic approaches to localization are employed. A solution to this problem can be to use a technique to compensate the dispersion, but this is an ill-posed problem because in order to compensate the dispersion, the position of the source, that is the unknown of the problem, is needed.

Nonetheless, dispersion is adding some extra information to the signal. This information can be exploited to estimate the distance traveled by the wave: the novel approach presented in chapter 5 is based on this concept. To apply this method, a deep knowledge about the properties of the material is needed. For this reason, we performed a preliminary study of dispersion inside the specimens, that can be used also as a guideline to calibrate the tangible interface when used with materials that differ from those employed here.

4.1 Dispersion curves measurement

As seen in section 3.4, dispersion curves can be computed when the elastic properties and the dimensions of a plate are known. The elastic properties of the materials are rarely available, unless specific materials with data sheets are employed. This type of materials might be rather expensive, and the objective of this research is also to obtain a cheap interface, that can eventually be set up by the user, for example by applying sensors on a table. When using "everyday materials" nothing is known about their properties. Moreover, they
can change from specimen to specimen, even if they are made of the same material. For these reasons, before starting to work on the actual process of localization and tracking, we measure the elastic properties (and dispersion curves) of the available specimens.

**Acquisition system setup**

To measure the dispersion curves, we set up a simple acquisition system (see figure 4.4). On the specimen to be analyzed we apply one actuator (A) and two receivers (R1 and R2). The actuator is a round piezoelectric diaphragm of diameter 2 cm: its function is to produce a predetermined, standard vibration in the material. As receivers we use two BU-1771 accelerometers.

A partly tricky operation when using contact sensor, is how to attach them to the object. For BU-1771 accelerometers the manufacturer (see [14]) suggests to use double-sided tape if the signal to be measured is up to 5 kHz, while use epoxy adhesives for higher frequencies. A study about this argument can be found in [12] and [11], where the authors measured the response of a sensor for various mount techniques: the best results are achieved with a screw mount, that is not feasible in our case, but very good results can be also reached with double sided tape or adhesives, up to the frequencies that we consider here (see Fig. 4.1). To fix the actuator to the surface we use normal tape on the top of it, because the use of double sided tape would introduce too much attenuation to the already weak vibration produced by the diaphragm (Fig. 4.5). The two accelerometers are attached with double sided tape.

A series of good advices on how to perform measurements can be found again in [11] and [12]:

- Tape the cable of the sensor to the structure, because it might interfere with the measurements process with its inertia.
- Power the sensor properly (about this, see [14], where a few different configurations to connect the BU-1771 accelerometers are described).
- Use the right acquisition and elaboration system with adequate resolution, sensitivity and range. Sampling frequency should be at least 5
times larger than the maximum frequency that has to be measured in order to avoid aliasing.

- Do not overload the system. The mass of the sensor should not be more than 10% the mass of the object to be measured.

Regarding power supply (point 4.1), a connection box has been previously built in our laboratory. This box (Fig. 4.2) has 8 mini-jack stereo inputs that can supply power at need (but they can be used also with sensors that do not necessitate of a power supply) and 8 RCA outputs. The box has also the purpose to adapt the impedance of the sensors to the impedance of the sound card used to digitalize the signals.

To record and sample the signals coming from the sensors, we needed a soundcard with advanced characteristics, if compared with standard PC devices (as suggested in point 4.1). We chose the Terratec EWS 800T for a few reasons:

- 8 RCA inputs/outputs in order to record simultaneously 8 different channels.
- 96 kHz maximum sampling frequency (more than 5 times the maximum frequency to be analyzed).
Figure 4.2: Box for impedance adaption and sensors power supply, with connected sensors.

Figure 4.3: Terratec EWS 800T soundcard.

- 24 bits resolution.
- Low latency time.

This acquisition system is used also later for localization and tracking experiments (see chapter 5). For more advices on how to do reliable measurements see also [15] and [20].

The three sensors are aligned on the same direction, at distances $d_1$ and $d_2$, as shown in Fig. 4.4. The actuator is connected to the output of a PC soundboard and amplified with an external amplifier to raise its level.
First of all, distances $d_1$ and $d_2$ have to be chosen. Distance $d_1$ must be small enough not to disperse too much the signal arriving at sensor $R_1$. In fact, the dispersion is evaluated in the tract between $R_1$ and $R_2$, so the signal at the first sensor, used as a reference, has to be as clean as possible. On the other hand, $d_1$ must be large enough to avoid interferences from vibrations transmitted from actuator in air. For these reasons, for MDF specimen we set $d_1$ to 3 cm for one group of measurements, 5 cm for a second group and 10 cm for a third group in order to verify that dispersion does not depend on the distance from the actuator. For plexiglass, these distances are reduced to 0.5 cm, obtaining good results as well (see Tab. 4.2).

For distance $d_2$, the value is chosen with a more accurate criterion. To obtain dispersion curves, we have to measure phase differences between the two sensors at various frequencies. To have a more reliable measurement, the phase difference between signals received by sensors $R_1$ and $R_2$ should be smaller than $2\pi$. For this reason, $d_2$ has to be shorter than the longest wavelength present in the tests (one wavelength corresponds to a $2\pi$ phase
difference). An empirical wavelength can be obtained with a simple test. A sensor is fixed to the surface, while another one is kept free. A sinusoid with a very low frequency is transmitted to the specimen. With an oscilloscope set to visualize outputs from both sensors, the distance at which the two signals are in phase is found by moving the free sensor alternatively close and far from the fixed one. The approximate wavelength found for MDF is around 8 cm, while for plexiglass the operation reveals to be harder than expected because of the strong impact of reflected waves at the edges. Thus, we keep distance $d_2$ below 8 cm, in particular 3 cm and 5 cm respectively for two sets of measurements for MDF, and 1 cm for plexiglass.

Next step is the choice of a signal to be propagated in the plate. We have run a few tests with different signals: a continuous sinusoidal sweep has been tested first, with frequencies ranging from 1000 Hz to 8000 Hz (these signals are commonly used to measure the frequency response of electroacoustic speakers). This choice hasn’t shown good results, because the received signal suffered too much from the perturbations caused by reflections. It has been then decided to use a discrete sweep, made with short sinusoidal bursts at increasing frequencies with larger frequency steps. The results have shown to be better than those obtained with the first method, but in frequency domain the bursts have resulted distorted. Finally, a discrete sweep with raised cosine windowed sinusoidal impulses has been employed (see an example in Fig. 4.6). This solution allows to have a single frequency impulse with a smoothly raising amplitude that avoids distortions.

The choice for $t_1$, the impulse length, and $t_2$, the distance between impulses, needs also a few comments. If the specimen had infinite width and length, $t_1$ might have been of any length, but because of edge’s reflections its length has to be limited. To avoid superimposition of reflected signals over direct signal, $t_1$ must be short enough to end before the first reflection reaches the sensor. This is directly related to the propagation speed and the geometry of the plate (Fig. 4.7). If group velocity is $v_g$, calculated as explained in section 3.4, then $t_1 < (d_r - d_d)/v_g$, where $d_d$ and $d_r$ are respectively the distance traveled by direct wave and first reflected wave. For the MDF panel the result is 2.1 ms, while for plexiglass is 1 ms. The value of $t_2$ can be fixed in a conservative way in 1 s to make separation of the different impulses easier.
Figure 4.6: Example of raised cosine windowed sinusoidal impulses: $t_1$ is the impulse’s length, $t_2$ is the distance between impulses.

We choose the frequency band on which to perform the discrete sweep considering the characteristics of both actuator and receiver. A lower limit around 1500 Hz is imposed by the poor frequency response of the piezoelectric diaphragm at low frequencies, while an upper limit at 5000 Hz is imposed by the resonance frequency of accelerometers BU-1771 (around 10000 Hz). For plexiglass measurements, after a few tests, we extend the band up to 8000 Hz with good results. The frequency steps can be set to 250 Hz, that allow a good resolution while reducing the length of acquisition. We create the discrete sweep containing 4 impulses for each frequency, to be able to reduce noise effect during the analysis process by averaging the similar signals. It is important to point out that in this frequency range, only the first flexural Lamb mode ($a_0$) is excited: extensional plate waves are transmitted only at higher frequencies due to their small out-of-plane motion at lower frequencies. This makes the study of phase velocities easier, because only one mode is present (see section 3.4).

A final factor to be taken into account is the attenuation. In these materials it is frequency dependent so, different frequencies saturate accelerometers in different ways. In particular, higher frequencies experience lower attenua-
tion, and consequently they clip the sensors while lower frequencies are barely detectable. For this reason, the amplitude of the sweep can be shaped with an attenuation, estimated with simple measurements, in 12 $dB/\text{decade}$ for MDF and 8 $dB/\text{decade}$ for the two plexiglass specimens. These values are not exact but they work to avoid clippings for both materials. The attenuated discrete sweep can be seen in Fig. 4.8.

To sum up, with the values calculated in previous paragraphs, and with a sampling frequency $f_s = 96 \, kHz$, the length of window for MDF is 200 $samples$, while for plexiglass we run two tests with length 100 and 50 $samples$, respectively. Distance between two consecutive impulses is 96000 $samples$. In Tab. 4.1 and 4.2 you can find a summary of all the performed tests.

Data analysis

With the acquired data, phase velocity can be calculated. S. Foti, in [18], proposes a complete method to compute phase velocities, the SASW method (Spectral Analysis of Surface Waves), used normally to analyze soil properties.
Figure 4.8: Raised cosine windowed sinusoids sweep, attenuated by 12 dB/decade.

<table>
<thead>
<tr>
<th>Test #</th>
<th>$d_1$ (cm)</th>
<th>$d_2$ (cm)</th>
<th>Attenuation (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>3</td>
<td>12</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
<td>3</td>
<td>12</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>5</td>
<td>5</td>
<td>12</td>
</tr>
<tr>
<td>7</td>
<td>10</td>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>10</td>
<td>5</td>
<td>12</td>
</tr>
</tbody>
</table>

Table 4.1: Summary of dispersion tests run on MDF specimen ($t_1 = 200$ samples and $t_2 = 96000$ samples for all tests, $d_1$ and $d_2$ as defined in figure 4.4).

The part about acquisition described by Foti’s document is mostly followed in the present work, while the computational part is slightly modified. It is worth anyway to briefly describe the SASW method.

To compute phase velocities, a sinusoidal impulse is transmitted through the soil and recorded by two sensors, separated by a distance $d_2$. Let the
Table 4.2: Summary of dispersion tests run on 2 different plexiglass specimens, one 5 mm thick, other one 2 mm thick (d₁ and d₂ as defined in figure 4.4, t₁ in figure 4.6).

<table>
<thead>
<tr>
<th>Test #</th>
<th>d₁</th>
<th>d₂</th>
<th>t₁</th>
<th>Frequency range</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.5 cm</td>
<td>1 cm</td>
<td>50 samples</td>
<td>2000-8000Hz</td>
</tr>
<tr>
<td>2</td>
<td>0.5 cm</td>
<td>1 cm</td>
<td>100 samples</td>
<td>2000-8000Hz</td>
</tr>
<tr>
<td>3</td>
<td>0.5 cm</td>
<td>1 cm</td>
<td>100 samples</td>
<td>1500-5000Hz</td>
</tr>
</tbody>
</table>

Signal at sensor **R1** be \(y₁(t)\) and signal at sensor **R2** be \(y₂(t)\). The first step is to compute the Fast Fourier Transform of the two signals:

\[
Y₁(\omega) = FFT(y₁(t)) \tag{4.1}
\]
\[
Y₂(\omega) = FFT(y₂(t)) \tag{4.2}
\]

Now the Cross Power Spectrum \(G_{y₁,y₂}\) of the two signals is calculated:

\[
G_{y₁,y₂} = Y₁(\omega) * Y₂(\omega) \tag{4.3}
\]

From the Cross Power Spectrum, the value for phase time delay \(\tau(\omega)\) is extracted:

\[
\tau(\omega) = \angle(G_{y₁,y₂})/\omega \tag{4.4}
\]

where operator \(\angle\) computes the phase response of Power Spectrum. Finally, from the time delay and by knowing the distance traveled by the wave, \(d₂\), velocity can be computed with the well known formula:

\[
v_{ph} = d₂/\tau(\omega) \tag{4.5}
\]

A different method to obtain phase velocities is used here. First we crop the various impulses recorded by the two sensors, maintaining the time coherence between them, in order to cut away as many reflections as possible. For this purpose, the starting point of cropping is set at the beginning of the impulse received by sensor **R1**, that is taken as reference. From this point we take a part of signal a few samples longer than the test impulse itself, in order to obtain the complete direct impulse also from sensor **R2**, that is delayed.
We then compute the FFT of the two signals obtained by averaging the four consecutive impulses at the same frequency. The phase response of the two signals is extracted from the Fourier transforms and the phase difference between the two signals is computed at the already known frequencies of interest, since all the impulses are sinusoids at a predetermined frequency. For example, if the frequency of the sinusoid contained in impulse \( k \) is \( f_k \), the operation to compute the phase difference, \( \delta_{\text{ph}}(f_k) \) [rad], is:

\[
\delta_{\text{ph}}(f_k) = \angle Y_2(f_k) - \angle Y_1(f_k)
\]  

(4.6)

This is performed for each frequency contained in the discrete sweep. It is then possible to compute the phase velocity for each inspected frequency with the formula:

\[
v_{\text{ph}}(f) = \frac{2\pi f d_2}{\delta_{\text{ph}}(f)}
\]  

(4.7)

Another useful value that can be computed is the time delay curve (in samples) between the signals received by the two sensors:

\[
\delta_{\text{samples}}(f) = f_s \frac{d_2}{v_{\text{ph}}(f)} = \frac{f_s \delta_{\text{ph}}(f)}{2\pi f}
\]  

(4.8)

To produce a final dispersion curve regarding each material, we compute the curves for various measurements and average them. A comment on the averaging process is needed:

- For measurements where \( d_1 \) and \( d_2 \) are equal, the averaging has to be done directly over the signals (e.g. averaging over the four consecutive impulses at the same frequency) in frequency domain.

- For measurements where \( d_1 \) is different but \( d_2 \) is the same, and for measurements where \( d_1 \) and \( d_2 \) are both different, \( \delta_{\text{ph}} \) has to be averaged.

This is done in order to reduce at minimum the propagation of measurement errors.
Figure 4.9: Measured phase velocity (dispersion) curves for MDF based on the not attenuated sweep.

**Measured dispersion results**

By plotting phase velocities towards the respective frequency, dispersion curves can be visualized.

For the MDF panel, the measurements from different sets resulted in very similar curves: it is then possible to affirm that dispersion is not dependent on the relative distance at which sensors are placed. For this reason, in the experiments run on the plexiglass plates we used only one value for $d_1$. In Fig. 4.9 the phase velocity curves (representing dispersion curves) based on the not attenuated sweep are plotted. On the other hand Fig. 4.10 shows the curves based on the sweep attenuated by 12 dB/decade. For the curves averaging process see paragraph 4.1. At higher frequencies the results are less linear. This can be explained by the fact that at higher frequencies waves travel faster: reflections arrive earlier and the direct signal is more affected by them.

In figure 4.11 the two averaged curves, that appear to be very similar, are plotted together with a 2nd order polynomial fitting of the overall averaged curve. The fitting is very good and we are allowed to say that the measurements are reliable.
Figure 4.10: Measured phase velocity (dispersion) curves for MDF based on the 12 dB/decade attenuated sweep.

Regarding plexiglass specimens, let us first consider the thick panel (6 mm). We can note that the curves regarding plexiglass differ from the ones regarding MDF because we used different signals to excite these plates (see Tab. 4.1 and Tab. 4.2). Figure 4.12 shows how the averaged curves obtained by using short impulses (50 samples) and longer impulses (100 samples) can be compared. We can note how at higher frequencies the short impulses give worse results than the longer ones: for this reason we consider these measurements outliers and we do not include them in the averaged results. An explanation to this phenomenon might be that the frequency of the cosine used to shape the window is higher for shorter windows. This higher frequency fell probably in the non linear band of the sensor’s frequency response. The curves regarding plexiglass appear to be less smooth if compared to the ones regarding MDF probably because of higher reflection’s impact. We must also keep in mind that the frequencies span used for plexiglass experiments is much larger than the one used for MDF. We can also note that the phase velocity is lower for plexiglass if compared with MDF. The results regarding experiments on plexiglass with frequency span from 1500 Hz to 5000 Hz are not plotted because in accordance with the other results.
The same observations made for the thick plexiglass panel can be done also for the thin one (2 mm) by observing Fig. 4.13. At high frequencies the sweep with 50 samples impulses is even less linear than the previous case regarding the thick panel. On the other hand the 100 samples test gives very good results if compared to the fitted curve. It is thus more correct to use the 100 samples curve as final result rather than the curve obtained by averaging both sets. It can be noted that the phase velocity is lower in the thin plexiglass than in the thick one (they are anyway of two different types of plexiglass).

4.2 Estimation of the material’s elastic properties

To calculate theoretical dispersion curves, the material’s elastic properties are needed. As we saw in section 3.4, the dispersion is related to the velocities of $S$ and $P$ wave modes. To extrapolate these two values from the acquired data, a minimization on the difference between calculated and measured values can be done. In other words, if $v_{ph,calc}$ and $v_{ph,meas}$ correspond to calculated and measured antisymmetrical mode phase velocities (in fact only this mode
Figure 4.12: Measured phase velocity (dispersion) curves for thick plexiglass panel.

propagates in our case), then we can define the data residual $R_d$ as:

$$R_d(v_\alpha, v_\beta) = \| v_{ph, meas} - v_{ph, calc}(v_\alpha, v_\beta) \|^2$$ (4.9)

This is a 2-D function that can be minimized by a search grid technique. Anyway, this task is rather difficult because of the flatness of the function caused by the low number of measured values and the limited frequency range. Therefore, we have to fix other constraints in order to make the minimization feasible with good accuracy.

Viktorov [26] and Tucker [25] suggest to constrain the solution of the minimization problem by fixing the value of Poisson’s ratio $\nu$ of the material. The $\nu$ value associated with different materials can be found in literature: $\nu$ confidence interval regarding MDF is about $0.2 \div 0.3$, while it is about $0.3 \div 0.5$ regarding plexiglass. By fixing the Poisson ratio, the residual becomes a 1-D function, that can be easily minimized to obtain either $v_\alpha$ or $v_\beta$. An example of this 1-D function (in relation to $v_\alpha$) is plotted in Fig. 4.14 for three meaningful values of $\nu$ (this is MDF case).

The minimization problem is now simple and if we choose for MDF $\nu =
0.25 and for both plexiglass specimens $\nu = 0.4$, the estimated elastic properties of the boards are:

- MDF: $v_\alpha = 2900\, m/s$, $v_\beta = 1600\, m/s$;
- Thick plexiglass: $v_\alpha = 2300\, m/s$, $v_\beta = 1000\, m/s$;
- Thin plexiglass: $v_\alpha = 1900\, m/s$, $v_\beta = 750\, m/s$;

Once the elastic properties of the board (in this case $v_\alpha$ and $v_\beta$) are estimated we can calculate the dispersion curves for the two lowest order modes for different ranges of frequencies by using the thin plates wave propagation theory (appendix A.3) and the formulas in section 3.4. These curves, shown in Fig. 4.15, Fig. 4.16 and Fig. 4.17 for the different analyzed specimens are in good agreement with the previously measured curves and those calculated by Viktorov (Appendix A) and Tucker ([25]).
Figure 4.14: Data residual as a function of the P-wave velocity for three values of $\nu$ (MDF case).

Figure 4.15: Phase velocity curve for $a_0$ mode of Lamb waves in MDF panel.
Figure 4.16: Phase velocity curve for $a_0$ mode of Lamb waves in thick Plexiglass panel (6 mm).

Figure 4.17: Phase velocity curve for $a_0$ mode of Lamb waves in thin Plexiglass panel (2 mm).
Chapter 5

Localization and tracking

The issue of creating a Human - Machine interface, in the classical sense of its conception, imposes to study two kinds of interactions: impulsive (tapping) and continuous (scratching). With these two kinds of interactions, we can generally perform any operation on a computer (see as an example the mouse, with click and drag).

A system to localize taps on the surface of an isotropic material’s plate has been developed at the Politecnico di Milano using the TDOA approach (see chapter 2.1). The next step is to track a continuous interaction on the same surface in order to create a complete interface.

The procedures to localize and track acoustic sources with several sensors can be divided into two steps: the estimation of the relative distances ($\Delta d$), that are the difference between the distances from the source to the various receivers, and the estimation of the position of the source from these relative distances. To complete each step, several methods are available: in this chapter we present only the ones tested for the purposes of this research.

5.1 Tap localization

Direction of Arrival Estimation

A system that we tested in order to detect the tap point, is based on a completely different concept because it does not rely on the estimation of relative distances. It employs 2-directional accelerometers. This type of accelerom-
Figure 5.1: Direction of arrival estimation with amplitude ratio (R1 and R2 are the receivers, S is the signal’s source).

Parameters sense the accelerations in the x and y directions, while usually the 1-directional ones sense the accelerations in the z direction. Suppose that a vibration reaches the accelerometer: let us call the sensor’s output component in the x direction $s_x(t)$ and in the y direction $s_y(t)$. Since the vibration is sensed in the two directions at the same point on the surface, the two signals are affected by the same dispersion. To evaluate the vibration’s direction of arrival, it is possible to use the ratio between the amplitudes of $s_x(t)$ and $s_y(t)$.

If the interaction is a tap, the peak values of the impulse, $A_x$ and $A_y$, can be measured. Their ratio gives the value of the tangent of the arrival’s direction angle, $\alpha$ (Fig. 5.1). In other words:

$$\alpha = \arctan \left( \frac{A_y}{A_x} \right)$$

By computing the angles $\alpha$ for sensor R1 and $\beta$ for sensor R2, and finding the intersection point of the two lines, the coordinates for source point S can be easily found.

The limit of this method, that we briefly tested, lies in the very high dependency on reflections that are summed to the direct signal: the amplitude
of the detected signal is not the one of the direct signal.

**TDOA ([22])**

A classic method employed for tap localization, used also for localization of acoustic sources in the air, is the TDOA (Time Delay Of Arrival). It is based on the arrival time of the signal to the various receivers. For this reason, we need a velocity value in order to calculate a relative distance starting from a time value.

To estimate the time delay of arrival, several techniques are available. Among them, two are normally employed, and are briefly described here:

- **Cross-correlation**

  A way to perform delay estimation is to find the peak of the cross-correlation function of two signals. In general terms, we can express the signals received by two sensors as:

  \[
  x_1(t) = s_1(t) * h_1(t) + n_1(t) \quad (5.2)
  \]

  \[
  x_2(t) = s_2(t) * h_2(t) + n_2(t) \quad (5.3)
  \]

  where \( s_1(t) \) and \( s_2(t) \) are the signals acquired by the receivers, convolved with their impulse responses \( h_1(t) \) and \( h_2(t) \), respectively. \( n_1(t) \) and \( n_2(t) \) are the noise signals in the two channels. Let us suppose that the two signals are the same except for a delay \( \Delta t \) and for an attenuation \( A \):

  \[
  s_2(t) = A s_1(t - \Delta t), \quad 0 < A < 1 \quad (5.4)
  \]

  Let us suppose also that the two receivers are the same \( (h_1(t) = h_2(t)) \) and that the noise in the two channels are not correlated, then the cross-correlation function becomes:

  \[
  c_{1,2}(\tau) = \int_{-\infty}^{+\infty} x_1(t)x_2(t + \tau) d\tau \quad (5.5)
  \]

  and the peak of this function can be found in correspondence of the delay \( \Delta t \).
Anyway, in the case of propagation in solids, the hypothesis in 5.4 is not verified because of the dispersion’s effect.

- **First arrival detection**

A more intuitive way to estimate time delays is to measure directly the time elapsed between successive signal arrivals. Let us define the starting point of the signal. Let us suppose that no perturbations are propagating in the material: the sensor’s output is the background noise, that is below a certain level. When a vibration starts to propagate in the medium, the output’s level of the various sensors raises, generally at different time instants, over the noise level. We can then measure the instant at which the signals reach the sensors by accurately defining a threshold level, that must be slightly higher than the noise level, but not too low in order to avoid spurious events. The time delays are measured starting from the arrival of the signal to the sensor closer to the source.

This method works properly when the first arrival is characterized by a concentrated peak (like an impulse). It is important to stress a major drawback when this method is used with dispersive materials: the effect of dispersion is that of spreading the wave front of the signal (see Fig. 5.2). For this reason the peak of an impulsive signal is reduced in am-
plitude and spread in time, introducing errors in the estimation of the arrival time.

It is very important to point out that TDOA cannot be used to track a continuous signal, because there are no wave fronts to threshold, except for the first arrival.

We can compute a relative distance by multiplying the time difference just obtained with the wave speed:

\[ \Delta d = v \Delta t \]  

(5.6)

where \( v \) is the wave speed. We pointed out in section 3.4 that wave speed is sometimes a hard concept to define. In this case, the speed that has to be used is group velocity. This kind of speed’s estimation is susceptible to errors due to dispersion, for the same reasons concerning delay estimation. Another limit of TDOA when applied to dispersive materials is to consider the speed as a constant.

**RDOR (Relative Distance Of Receivers)**

**Note:** The following paragraph is a summary of the original version of this thesis: for copyright reasons, this part can not be published in its integrity as it is property of ISPG at Politecnico di Milano and Tai-Chi consortium.

TDOA employs concepts that are usually meant for source localization in the air, that is a non dispersive medium. For dispersive media, on the contrary, this technique still gives acceptable results, even though the precision is limited. To be able to have a more precise estimation of the source position, dispersion has to be taken into account. In order to make TDOA approach as close as possible to the in-air case, we can use, for example, a dispersion’s compensation filter. But this is an ill-posed problem: to create a filter to compensate the dispersion, the information about the space traveled by the signal (that is the unknown of the problem) is needed.

The phase displacement, introduced by the different phase velocities at various frequencies, increases proportionally with the distance. As an example,
Figure 5.3: Right-angled triangular array of sensors (S is the signal’s source, Ri are the receivers).

Suppose that frequencies $f_1$ and $f_2$ are traveling at phase velocities $v_{ph1}$ and $v_{ph2}$. After a distance $d$, the phase difference is:

$$\Delta \Phi = 2\pi d \left( \frac{f_1}{v_{ph1}} - \frac{f_2}{v_{ph2}} \right)$$

(5.7)

that is proportional to $d$.

The dispersion limits the accuracy in the TDOA approach, but it can be also considered an extra information about the distance traveled by the signal. Let us suppose to be in a 1-D situation and to produce a single frequency vibration ($f$). The signals at points $x$ and $x + d$ are respectively $s_x(t)$ and $s_{x+d}(t)$, where $d$ is unknown. If phase at point $x$ is set as reference, phase at point $x + d$ can be measured, and $d$ can be computed from the phase velocity value at frequency $f$.

In a 2-D situation, sensors are not placed on the same direction. Let us consider the setup presented in Fig. 5.3. We need to learn the relative distance between the reference ($R_0$) and the other sensors, rather than the actual distance traveled by all four signals from the source point to the four sensors. To compare the phase for each frequency and for each sensor to compute
relative distances is a very long operation. To circumvent the problem we have developed a different approach, that is the basic concept of the RDOR technique.

Let our attention be focused for example on sensor \textbf{R1}. We want to compute $\Delta d_1$, the relative distance from source of sensor \textbf{R1} and the reference sensor (\textbf{R0}), that we define, for each sensor $i$, as:

$$
\Delta d_i = d_0 - d_i
$$

(5.8)

It is possible to create a filter using the dispersion curve of the material, that "inverse propagates" the signal at sensor \textbf{R1} to sensor \textbf{R0}. The inverse propagation compensates the dispersion effect and also the time delay between the two signals. Since the filter is function of the relative distance, the optimum filter is obtained when it is calculated using the exact relative distance $d_1$. The RDOR method is able to compute the correct $d_1$ by inverse propagating the signal at sensor \textbf{R1} with a series of filters computed for different distances and by comparing the results with the reference signal. The best match corresponds to the optimum filter and consequently to the correct relative distance. The process is then applied to each sensor in order to obtain the $d_i$.

The main advantage of RDOR over the other techniques is to take dispersion into account: for this reason, both limits of the TDOA method (the assumption of a constant speed and the spreading of the wavefront) are circumvented.

5.2 Scratch tracking with RDOR

The first task required by any Human - Computer interface, the tap localization, has been largely studied. The task of scratch tracking is still an open problem, that, as we have seen, cannot be solved with TDOA technique, because of the non-impulsive nature of a continuous signal. A solution to estimate time delays could be the cross-correlation method, but dispersion limits its employment. I propose here to use the RDOR also for source tracking; in fact, this method can work with any kind of signal and is not affected by
Let us suppose to be in the situation depicted in Fig. 5.4.

The signal received by any of the four sensors might be the one shown in Fig. 5.5.

At the time instant $t_1$, an object, that might be for example the tip of a finger or a pen, starts to scratch the surface of the interface in position $[x(t_1), y(t_1)]$: the distance from this point to sensor $R_i$ is $d_i(t_1)$. At the instant $t_2$, the tip will be in position $[x(t_2), y(t_2)]$ at a distance $d_i(t_2)$ from the sensors, and so on. Let the time instants be equally spaced by $\Delta t$. In this way, the signal $s_i(t)$ acquired by sensor $R_i$ can be divided into windows. Let $w_{i,1}(t)$ be the window starting at time $t_1$ and ending at time $t_2$ for sensor $R_i$. Let then $w_{i,2}(t)$ be the window starting at time $t_2$ and ending at time $t_3$, and so on for all the following time instants. If the window is sufficiently short we can suppose that the tip will not move too far. So, RDOR technique can be applied to the windowed signals $w_{i,k}(t)$. The relative distances:

$$\Delta d_i(t_k) = d_0(t_k) - d_i(t_k)$$

(5.9)
can be computed for the $k$th window, and then the approximate position at instant $t_k$ can be derived with several methods (see section 5.3). By a point-by-point trajectory interpolation, the movement of the tip can be tracked.

Now we must add an observation regarding the feasibility of the operations described here. The filtering and successive comparison of various signals involved in the process are operations with a high computational cost. The resolution of the $\Delta d$ estimation with RDOR is related to the number of tested distances and therefore to the number of filters. The dimensions of the surface define the interval of values that $\Delta d$ can assume. To make this technique available to real time applications I have developed a series of optimizations (see section 5.4).

5.3 Source position estimation

Many localization techniques, like the ones employed and developed in this project work, are based on the estimation of relative distances. There are a few methods to calculate the coordinates of the source from these relative distances. In this work, we tested two of them: a geometrical solution based
Figure 5.6: Source position estimation in 1D situation (Ri are the receivers, S the signal’s source).

Figure 5.7: Source position estimation in 2D with two sensors generates an infinite number of solutions lying on an hyperbola (S is the source, Ri are the receivers).

on hyperbolas intersection (see [21]) and a numerical solution based on an inverse problem formulation.

**Hyperbolas intersection point (Tobias,[24])**

Consider the 1-D case in Fig. 5.6. If the distance between the two sensors (R1 and R2) is known ($D = d_1 + d_2$), and the relative distance from source S to the two receivers ($\Delta d = d_2 - d_1$) is estimated, then the position of the source is easily and uniquely determined by solving this system of equations:

\[
\begin{align*}
    d_1 + d_2 &= D \\
    d_2 - d_1 &= \Delta d
\end{align*}
\]  

(5.10)

In the 2D situation, on the other hand, when the source lies at a point in a
Figure 5.8: Regions with ambiguous solutions exists near and behind each sensor.

plane, the difference in distance traveled by the wave to a pair of sensors ($\Delta d$) generates an infinite number of possible solutions, which lie on an hyperbola, with the sensors in the two foci (hyperbola is defined as the set of all points in the plane the difference of whose distances from two fixed points, called foci, is constant, see Fig. 5.7). The use of one or more additional sensors permits to find the source coordinates by intersection of two or more hyperbolas, defined by relative distances of other sensor pairs. In general, a number of $N$ receivers yields $N - 1$ relative distances and coordinates. Thus, two is the minimum number of transducers required for the linear location (1D case), three for a plane and four for a volume.

It has to be pointed out that sometimes ambiguous solutions arise when the minimum number of sensors is used. In the region close to and "behind" each sensor, there is a certain area in which twin solutions occur. This is shown in Fig. 5.8. Both solutions are physically meaningful and in order to resolve the ambiguity, additional information must be collected. This can be done by measuring the $\Delta d$ to an extra transducer and comparing it with the
calculated value for the source location.

The algorithm used to calculate the source location from collected data may be based on several methods. Numerical solutions can be calculated using iterative methods, tables or a direct mathematical solution. Tables have a limited resolution and neither tables nor iterative methods can generally be used to correctly resolve situations where ambiguous solutions occur. Thus, the best way to apply this method is to find an exact mathematical algorithm: Tobias [24] found one for a plane surface, as in our case, for an array of four transducers, where one of them is used to avoid ambiguities. It is important to note that this method is independent from the position of the sensors, but some configurations give better results than others and are more robust against ambiguities. Tobias discussed ambiguity and spatial resolution for three different arrays of sensors (see [24]).

Let us consider a right-angled triangular array of sensors, as in figure 5.3, where sensors are arranged at the cartesian coordinates \((0,0)\), \((x_1, y_1)\) and \((x_2, y_2)\). Sensor \(R_0\) in this case is taken as reference. Let \(\Delta d_1\) be the relative distance between and \(R_1\) and \(\Delta d_2\) between \(R_0\) and \(R_2\). These two values define two hyperbolas. The source location \((x,y)\) is then given by:

\[
(x_1^2 + y_1^2) - \Delta d_1^2 = A_1 \tag{5.11}
\]

\[
(x_2^2 + y_2^2) - \Delta d_2^2 = A_2 \tag{5.12}
\]

If \(A_1 x_2 - A_2 x_1 \geq 0\) then \(\tan^{-1} \frac{A_1 y_2 - A_2 y_1}{A_1 x_2 - A_2 x_1} = B\) \tag{5.13}

If \(A_1 x_2 - A_2 x_1 < 0\) then \(\tan^{-1} \frac{A_1 y_2 - A_2 y_1}{A_1 x_2 - A_2 x_1} + \pi = B\) \tag{5.14}

\[
\cos^{-1} \frac{A_2 \Delta d_1 - A_1 \Delta d_2}{\sqrt{(A_1 x_2 - A_2 x_1)^2 + (A_1 y_2 - A_2 y_1)^2}} = C \tag{5.15}
\]

From these relations, we can compute two solutions in polar coordinates for source position \((r_1, \theta_1)\) and \((r_2, \theta_2)\):

\[
r_1 = \frac{A_1}{2(x_1 \cos \theta_1 + y_1 \sin \theta_1 + \Delta d_1)} \quad \text{where} \quad \theta_1 = B - C \tag{5.16}
\]
and
\[ r_2 = \frac{A_2}{2(x_1 \cos \theta_2 + y_1 \sin \theta_2 + \Delta d_2)} \quad \text{where} \quad \theta_2 = B + C \quad (5.17) \]

If either \( r_1 \) or \( r_2 \) is negative, then that solution is invalid.

The corresponding cartesian coordinates are given by the well-known:
\[ (x, y) = (r \cos \theta, r \sin \theta) \quad (5.18) \]

It might happen that both values of \( r \) are positive: in this case the two solutions lies in the areas close to or behind the sensors. To resolve the ambiguity we can use a fourth sensor, \( \mathbf{R3} \), at position \((x_3, y_3)\) with relative distance \(\Delta d_3\). The correct location can be found with:
\[
\frac{\sqrt{(x-x_3)^2 + (y-y_3)^2} - \sqrt{x^2 - y^2 - \Delta d_3}}{\sqrt{(x-x_3)^2 + (y-y_3)^2}} = D \quad (5.19)
\]
that has to be computed for both solutions of \((x, y)\) found above. The solution resulting in the smaller value for \(D\) should be preferred.

**Inverse problem formulation ([23])**

The estimation of the source position from the relative distances can be achieved by an inverse problem formulation. If we consider the same array of sensors as for Tobias method (Fig. 5.3), the problem can be formulated with the following elements:

- **Parameters (experiment configuration):** the position of the sensors, in this case four, \(S_i(x_i, y_i), i = 0, 1, 2, 3\).

- **Unknowns (model):** the position of the source \(m = [x, y]^T\).

- **Observed data (of cardinality \(N = 3\)):** differences \(\Delta d_i\) between the traveled distance from source to reference sensor, in this case supposed to be \(\mathbf{R0}\), and the traveled distance to the other sensors \((d_{obs} = [\Delta d_1, \Delta d_2, \Delta d_3]^T)\).
The distances $d_i$ can be defined as:

$$d_i = \sqrt{(x - x_i)^2 + (y - y_i)^2} \quad (5.20)$$

and consequently the relative distances (observed data) are:

$$\Delta d_1 = d_1 - d_0$$
$$\Delta d_2 = d_2 - d_0$$
$$\Delta d_3 = d_3 - d_0 \quad (5.21)$$

The link between model and observed data leads to a Jacobian matrix $G$, linearized around reference model $\mathbf{m}_0 = [x_0, y_0]^T$:

$$G = \begin{bmatrix} \frac{x-x_0}{d_0} & \frac{x-x_1}{d_1} & \frac{x-x_2}{d_2} & \frac{x-x_3}{d_3} \\ \frac{y-y_0}{d_0} & \frac{y-y_0}{d_1} & \frac{y-y_0}{d_2} & \frac{y-y_0}{d_3} \\ \frac{x-x_0}{d_0} & \frac{x-x_1}{d_1} & \frac{x-x_2}{d_2} & \frac{x-x_3}{d_3} \end{bmatrix} \quad (5.22)$$

and to the linear system:

$$\begin{bmatrix} \Delta d_1 \\ \Delta d_2 \\ \Delta d_3 \end{bmatrix} = G \cdot \begin{bmatrix} x \\ y \end{bmatrix} \quad (5.23)$$

which can be solved for the model $\mathbf{m} = [x, y]^T$ to find source position coordinates. The forward problem is non-linear and the solution can be determined with a good precision by using an iterative inversion procedure like the Tarantola’s technique for non-linear inverse problems (appendix B.5). To find the solution the following iterative fix-point algorithm has been implemented:

$$\mathbf{m}_{k+1} = \mathbf{m}_{pr} - (G_k^T C_d^{-1} G_k + C_{M}^{-1})^{-1} G_k^T C_d^{-1} [(g(\mathbf{m}_k) - d_{obs}) - G_k (\mathbf{m}_k - \mathbf{m}_{pr})] \quad (5.24)$$

where $\mathbf{m}_{pr}$ and $C_m$ are respectively the mean and the covariance matrix of the apriori model, $d_{obs}$ is the vector of observed data, $C_d$ is the covariance matrix of the measured uncertainties and of the modelling errors, $G_k$ is the linearized Jacobian matrix at iteration pass $k$, $\mathbf{m}_k$ and $\mathbf{m}_{k+1}$ are the vectors of model at $k$th and $(k + 1)$th iteration. The iteration stops when a fixed maximum
number of iteration is reached or when:

\[ |m_{i,k+1} - m_{i,k}| < \varepsilon \quad \forall \quad i = 1, \ldots, M \]  

(5.25)

For the experiments run in this work, the number of iterations is usually 3-4.

5.4 RDOR optimizations

Note: The following paragraph is a summary of the original version of this thesis: for copyright reasons, this part can not be published in its integrity as it is property of ISPG at Politecnico di Milano and Tai-Chi consortium.

If implemented in practice, the RDOR and the tracking technique are very resource-demanding. For this reason I have studied some optimizations to make the algorithm available for real time applications. In the chapter contained in the original version of this thesis, some implementation details are described, that have to be omitted here.

Resolution optimization for tap localization

When we employ the RDOR to estimate \( \Delta d \), the spatial resolution, proportional to the number of filters employed, can generate a problem of resources availability: if we run a pure exhaustive search for all the possible \( \Delta d \), for example, on a square surface with 40 cm long diagonals, and we require a spatial resolution of 5 mm, then the number of filters will be \( 40 / 0.5 = 160 \). If the filtering has to be done for three sensors, then we will require 480 filtering operations (Note: the maximum value that \( \Delta d \) can assume is half the distance between the sensor and the reference. The positive sign points that the source is closer to the reference sensor; on the other hand, the negative sign points that the source is closer to the sensor).

To avoid this problem we can employ a hybrid system, where a first estimation of the position is found with TDOA method, and then a refinement is performed by an exhaustive search with RDOR, only close to the point estimated with TDOA.
Tracking optimization

It seems to be hard to employ RDOR directly for source tracking: if an exhaustive search had to be done for every window, the computation time would raise indefinitely. It is possible, anyway, to make two observations:

- Points detected in consecutive windows are likely to be close to each other.
- There is always an impulsive part at the beginning of a continuous signal, caused by the initial touch of the user on the surface (see Fig. 5.9).

The first observation permits to reduce the number of filters for each window, by searching only close to the point detected in the previous window: the method becomes a sort of differential search. A higher resolution can be therefore achieved because the search is more focused.

To use this optimization, a starting point for the tracking is needed. We can find a starting point by considering the second observation. Using tap localization on the initial impulse, a starting point can be found, and the tracking algorithm can be initialized with that value.
A complete Human - Computer interface can now be developed: its scheme is presented in the following section.

5.5 Complete Human - Computer Interface

As we have seen, the two tasks required by a complete Human - Computer interface are localization and tracking. A few techniques have been described to accomplish both tasks. Anyway, before applying them, we need to distinguish between the two kinds of interaction in order to choose the correct algorithm to employ: a pattern recognition is needed. We propose here an easy scheme to solve this problem.

Signal’s type recognition

There are several techniques for pattern recognition, based on spectral analysis and time domain analysis. In the present situation, anyway, a quick and easy method is needed, not to make the algorithm more complex.

Because the tracking algorithm works on time windows, then the whole process has been divided into time blocks for simplicity.

Let us consider the time window $k$ of length $W$ samples. Inside this window, we can find one of the following types of signal (if $W$ is short enough):

- **Silence**: only background noise is present.
- **Impulse**: a tap happens inside the window.
- **Continuous**: the beginning, the end or a part of a continuous signal is contained in the window.

A way to recognize what type of signal is present can be to calculate a rough envelope, based on a very limited number of points. Some considerations about the signal follow. They are based on the knowledge of the previous windows situation, and on the slope of the envelope. A summary is presented in Tab. 5.1.

With this simple solution, the various windows can be routed to the appropriate algorithm for further analysis. Since signals arriving to different sensors are usually delayed one from another, it may happen that different
types of signal are present for the different sensors in the same window. The ambiguities can be avoided by choosing the most frequent type among all the signals.

**Complete HCI block scheme**

For these block schemes, we take into account the situation depicted in figure 5.4, with 4 sensors, but it can be extended to any number of sensors.

In figure 5.10 a high level block diagram of the complete system is shown. The continuous signal arriving from the sensors is collected in a buffer of size $W$. When the buffer is full, a window of length $W$ is transmitted to the pattern recognition box. This box returns current window’s signal type, using Tab. 5.1 and the information about previous window retrieved from the memory block. The new value is then stored for next iteration. This information is also used to route the windowed signals to the correct algorithm to be processed.

If the window contains silence, nothing is performed, and next window is analyzed.

If the window contains a tap, the signals pass to the TDOA block, that estimates a first position’s value. This information is transmitted, together with the signals, to the RDOR block, that refines the estimation starting from the position computed with the TDOA method.

If the window contains a continuous interaction, the signals are passed to the RDOR block, together with the position estimated in previous window, or at the previous tap, that is stored in the actual position memory. The estimated position can be filtered with a Kalman Filter, in order to reduce the effect of spurious detections that might deviate the trajectory and compromise the differential approach of tracking: the RDOR filters might detect,

<table>
<thead>
<tr>
<th></th>
<th>Silence</th>
<th>Impulse</th>
<th>Continuous</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flat</td>
<td>Silence</td>
<td>Silence</td>
<td>Silence</td>
</tr>
<tr>
<td>Encreasing</td>
<td>Continuous</td>
<td>Continuous</td>
<td>Continuous</td>
</tr>
<tr>
<td>Decreasing</td>
<td>Impulse</td>
<td>Silence</td>
<td>Silence</td>
</tr>
</tbody>
</table>

Table 5.1: Scheme for pattern recognition: in the first vertical column the signal’s envelope slope direction, in the first row the situation in the previous window.
Figure 5.10: Blocks diagram of the complete HCI: notice that the dotted boxes are memory blocks.

for example, to reflected signals instead of direct signals.

The pattern recognition can be also used to distinguish different operations related to different types of signals: for example the tap can act like a mouse click, while continuous trajectory can act like a mouse dragging.
Chapter 6

Experimental results

In this chapter we intend to present the results obtained with RDOR applied to localization and tracking problems.

6.1 Localization results

Localization test setup

To perform tap localization on both plexiglass and MDF specimens, we used a setup made up of five sensors. Four sensors have been positioned at the corners of a rectangle while the fifth sensor, used as reference, has been placed in the center (see Fig. 6.1) in order to obtain a reference signal as free from reflections as possible.

The MDF panel had dimensions 152 cm X 106 cm x 0.6 cm, while the sensors were placed at coordinates [136, 23], [136, 83], [16, 83], [16, 23] and [76, 53] (measured starting from the bottom left corner of the table). The tapping has been performed with a screwdriver’s tip. For each position tested on the plate, we performed four consecutive tappings and averaged them in order to reduce the noise effect.

The Δd_i have been estimated using the RDOR and searching exhaustively over the whole range of possible values, while the source position has been determined by solving the inverse problem with the Tarantola’s method (see chapter 5).
Comment results

MDF experiment A11

The configuration of the experiment is shown in Fig. 6.2; the real coordinates of the position of the finger touch (represented in the figure by a circle) are:

\[ x_T = 0.91 \ m, \ y_T = 0.455 \ m \]

while the solution of the inversion gives:

\[ \hat{x}_T = 0.912 \ m, \ \hat{y}_T = 0.448 \ m \]

The result has a good accuracy.

By using a Tarantolas approach it is also possible to show the posterior probability density of the touch, depicted in Fig. 6.3; the green circle represents the estimate position of the finger touch.
Figure 6.2: Tap localization result for experiment MDF A11 (Rx are the receivers, Tx is the source).

Figure 6.3: Solution of the experiment A11 posterior probability density of the touch.
The configuration of the experiment is shown in Fig. 6.4; the real coordinates of the position of the finger touch (represented in the figure with a circle) are:

\[ x_T = 0.91 \text{ m}, \quad y_T = 0.755 \text{ m} \]

while the solution of the inversion gives:

\[ \hat{x}_T = 0.919 \text{ m}, \quad \hat{y}_T = 0.777 \text{ m} \]

The result has a good accuracy; the posterior probability density of the touch is shown in Fig. 6.5; the green circle represents the estimate position of the finger touch.

A summary of tap localization tests run on MDF panel is shown in figure 6.6.
Figure 6.5: Solution of the experiment A32 posterior probability density of the touch.

Figure 6.6: Summary of tap localization experiments on MDF: crosses represent real coordinates while dots represent estimated coordinates.
6.2 Tracking results

In this section we present the results obtained only on the MDF panel: it has been noticed that, because of the extreme smoothness of plexiglass surface, the level of a scratch is practically impossible to sense because it is almost inaudible. For this reason, no results about this material are presented and a solution for this problem is left to further researches.

Tracking experiments setup

To perform these tests we employed the following setup (see figure 6.7):

- MDF panel of dimensions 152 cm X 106 cm X 0.6 cm, placed on a table with a layer of foam rubber between them.

- 4 BU-1771 sensors, placed at coordinates $R_0=[46,30.5]$, $R_1=[106,30.5]$, $R_2=[106,76]$ and $R_3=[46,76]$ (calculated in cm starting from the bottom left corner of the table).
Table 6.1: Scratch tracking tests summary (Start and End are coordinates in cm).

<table>
<thead>
<tr>
<th>Test #</th>
<th>Start</th>
<th>End</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>45,7.5</td>
<td>45,37.5</td>
<td>Vertical</td>
</tr>
<tr>
<td>2.2</td>
<td>30,7.5</td>
<td>30,37.5</td>
<td>Vertical</td>
</tr>
<tr>
<td>2.3</td>
<td>15,7.5</td>
<td>15,37.5</td>
<td>Vertical</td>
</tr>
<tr>
<td>3.1</td>
<td>45,7.5</td>
<td>15,7.5</td>
<td>Horizontal</td>
</tr>
<tr>
<td>3.2</td>
<td>45,22.5</td>
<td>15,22.5</td>
<td>Horizontal</td>
</tr>
<tr>
<td>3.3</td>
<td>45,37.5</td>
<td>15,37.5</td>
<td>Horizontal</td>
</tr>
<tr>
<td>4.1</td>
<td>45,7.5</td>
<td>30,22.5</td>
<td>Diagonal</td>
</tr>
<tr>
<td>4.2</td>
<td>45,37.5</td>
<td>30,22.5</td>
<td>Diagonal</td>
</tr>
<tr>
<td>4.3</td>
<td>15,37.5</td>
<td>30,22.5</td>
<td>Diagonal</td>
</tr>
<tr>
<td>4.4</td>
<td>15,7.5</td>
<td>30,22.5</td>
<td>Diagonal</td>
</tr>
</tbody>
</table>

- A small screwdriver to produce the continuous scratch on the panel’s surface.

We run several different test-sets, tracking the scratch in different parts of the panel and taking different directions. A summary of the coordinates of the starting and ending points used in the various experiments can be seen in Tab. 6.1. Note that for the position estimation, sensor R0 is taken both as reference and as origin of coordinates: the values in table are referred to it.

The continuous excitations have been produced by scratching the tip of a screwdriver on the surface of the panel. An important observation has to be done: the scratching has been performed manually, trying to keep a constant speed and a straight trajectory, but it is clear that the results are not as precise as an automated test might have been. For this reason, all the results about errors of estimation presented in this section have to be considered approximations, since the theoretical trajectory, that is the one to be compared with the estimated trajectory, is just an approximation.

**Analysis process**

The steps that lead us to the estimation of the screwdriver’s tip trajectory are:
1. **Signals recording**: the signals coming from the four sensors are recorded simultaneously, in order to maintain the time coherence, and sampled at a frequency $f_s = 96\ kHz$.

2. **Signals preprocessing**: the single scratches are isolated and band-pass filtered between $[500 \div 8000] Hz$, in order to eliminate distortions introduced by the power supply in the low frequencies range (50 Hz and some of its harmonics) and by the peak in the sensor’s frequency response in the high frequencies range.

3. **Windowing**: since the process is run offline, all the signals are previously divided into windows of length 2048 samples (but other tests have been run also for 1024 and 512 samples long windows), that at $f_s$ sampling rate corresponds to a time of 20 ms.

4. **Starting point $\Delta d_i(0)$ calculation**: to create the filters for the first window, we need the $\Delta d_i(0)$ to initialize them. Since these experiments are meant only for tracking algorithm’s testing, the starting point is fed to the tracking function as a known value and not estimated from the trailing tap (see section 5.4).

5. **Tracking**: the windowed signals and the starting point $\Delta d_i(0)$, together with the filters bank’s resolution, are fed to the tracking function, (see section 5.4). A few different resolutions have been tested, because we have to determine a compromise between speed of calculation, precision and resistance towards spurious detections. The best compromise for these sets of measurements has been found in a resolution’s range $[(\Delta d_i(k)−9\ mm)÷(\Delta d_i(k)+9\ mm)]$ with 3 mm steps. These increments depend also on the speed at which the screwdriver is moving. To estimate the source position we use Tarantola’s inversion method. A vector with the estimated positions at window $k$ is created: $[x_{est}(k), y_{est}(k)]$.

6. **Kalman Filtering**: for these sets of experiments, the Kalman filtering is postponed at the end of the tracking, only to smoothen the evaluated trajectory and not to eliminate spurious detections. The result is the vector $[x_{kal}(k), y_{kal}(k)]$. 

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7. **Error computation**: we create a vector \([x_{th}(k), y_{th}(k)]\) with the theoretical trajectory’s points, interpolated at constant intervals whose length is the total length of the trajectory divided by the number of windows, \(N_w\). We can then calculate the Mean Error of estimation, \(\bar{e}\):

\[
\bar{e} = \frac{1}{N_w} \sum_{k=1}^{N_w} \sqrt{(x_{est}(k) - x_{th}(k))^2 + (y_{est}(k) - y_{th}(k))^2}
\] (6.1)

The Standard Deviation of the error, \(\delta_e\), is also calculated.

8. **Plot results**: the tracking results together with the theoretical trajectory are plotted in a \(x - y\) plot.

9. **Starting point error effect**: since the complete system is based on the estimation of initialization point for tracking, we have conducted a brief study on the effect caused by an error in starting point’s estimation on the overall estimation error \(\bar{e}\). We have initialized the tracking algorithm with variable points at a distance from 1 cm to 5 cm around the correct one and we have computed and plotted the resulting overall estimation error towards the distance from the correct point.

**Commented results**

Only one test per set is presented here, the rest of the figures can be found in Appendix C.

**Set #2**

In the set number 2, three vertical trajectories have been produced with the screwdriver. In Fig. 6.8 we can compare the estimated and theoretical trajectories, while Fig. 6.9 represents the developing of error in time. We can explain the fluctuations of the error by considering the not constant speed maintained by the screwdriver, that was instead assumed to create the theoretical path.

In Tab. 6.2 a summary of mean error and standard deviation for set 2 is shown: notice that the mean error is always below 2 cm while the standard deviation is below 1 cm.
Figure 6.8: Tracking test 2.1: estimated and theoretic trajectories inserted in the panel’s usable area.

Figure 6.9: Test 2.1 corresponding error vs. window progressive number.
Table 6.2: Tests set 2: error values for estimated and smoothed trajectories (Kal) in mm (\( \bar{\tau} \) is the mean error, \( \delta e \) is the error's variance).

<table>
<thead>
<tr>
<th>Test #</th>
<th>( \bar{\tau} )</th>
<th>( \delta e )</th>
<th>( \bar{\tau}_{Kal} )</th>
<th>( \delta e_{Kal} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>15.4</td>
<td>7.0</td>
<td>14.8</td>
<td>6.6</td>
</tr>
<tr>
<td>2.2</td>
<td>20.8</td>
<td>8.6</td>
<td>20.0</td>
<td>8.9</td>
</tr>
<tr>
<td>2.3</td>
<td>15.6</td>
<td>9.6</td>
<td>14.3</td>
<td>8.7</td>
</tr>
</tbody>
</table>

Figure 6.10: Tracking test 3.3: estimated and theoretical trajectories inserted in the panel’s usable area.

Set #3

In set number 3, three horizontal trajectories have been produced with the screwdriver. In Fig. C.5 we can compare the estimated and theoretical trajectories, while Fig. C.6 represents the developing of error in time. The "jump" happening in the middle of the trajectory is probably caused by a cluster of spurious detections, that introduces a differential error that propagates in the continuation of the tracking. This error might probably be eliminated by using the Kalman Filter at every iteration and not in post-processing on the complete trajectory. Anyway, we can see from Fig. 6.11 that at the end of the trajectory the system partly recovers from the error.

In table 6.3 a summary of mean error and standard deviation for tests set
Table 6.3: Tests set 3: error values for estimated and smoothed trajectories (Kal) in mm ($\overline{e}$ is the mean error, $\delta_e$ is the error’s variance).

<table>
<thead>
<tr>
<th>Test #</th>
<th>$\overline{e}$</th>
<th>$\delta_e$</th>
<th>$\overline{e}_{Kal}$</th>
<th>$\delta_{eKal}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1</td>
<td>9.1</td>
<td>3.7</td>
<td>8.2</td>
<td>3.2</td>
</tr>
<tr>
<td>3.2</td>
<td>6.7</td>
<td>3.4</td>
<td>5.7</td>
<td>3.0</td>
</tr>
<tr>
<td>3.3</td>
<td>26.8</td>
<td>12.2</td>
<td>26.2</td>
<td>11.8</td>
</tr>
</tbody>
</table>

3 is presented. Figures in appendix C demonstrate that the trajectories in set number 3 are the worst tracked: this can be explained by noticing that the reference sensor is the farthest from the source, and for this reason it is more corrupted by reflections. We can solve this problem by actively selecting the reference sensor as the closest to the source at the beginning of the interaction.

**Set #4**

In set number 4, four diagonal trajectories have been produced with the screwdriver. In Fig. 6.12 we can compare the estimated and theoretical trajectories, while Fig. 6.13 represents the developing of error in time. In this case, we can see that the estimated and theoretical trajectories differ by a constant
value: the reason might be that when the screwdriver has been scratched, the trajectory to be followed has been mistakenly marked on the plate. In Fig. 6.14 we have zoomed on the trajectory to better understand the evolution of the tracked points: we can note that a small deviation from the straight path, probably caused by spurious detections, is recovered even without Kalman filtering.

In Tab. 6.4 a summary of mean errors and standard deviations for set 4 is presented: this is the best set if compared with the other two. These measurements scored a very low Standard Deviation, meaning that the trajectory is very smooth.

For set 4, we have tried to track the signal also with windows of length 512 samples: the result is slightly more unstable than the one for 2048 samples, mainly because the Fourier Transforms are performed on fewer samples. For this set, a range $[\Delta d_i(k) \div 5 \text{ mm} \div \Delta d_i(k) + 5 \text{ mm}]$ for the filters has been used. The results can be seen in Fig. 6.15 and 6.16.

On set 4.4 we have also tested the effect of starting point’s errors: it can be seen from Fig. 6.17 that with an error in initialization of about $3 - 4 \text{ cm}$, the resulting tracking is still acceptable (the corresponding errors are $22 - 28 \text{ mm}$).
Figure 6.13: Test 4.1 corresponding error vs. window progressive number.

Figure 6.14: Tracking test 4.3: closer look.
Table 6.4: Tests set 4: error values for estimated and smoothed trajectories (Kal) in \textit{mm} (\(\overline{\varepsilon}\) is the mean error, \(\delta_e\) is the error’s variance).

<table>
<thead>
<tr>
<th>Test #</th>
<th>(\overline{\varepsilon})</th>
<th>(\delta_e)</th>
<th>(\overline{e})Kal</th>
<th>(\delta_{e\text{Kal}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.1</td>
<td>10.9</td>
<td>2.4</td>
<td>10.3</td>
<td>1.2</td>
</tr>
<tr>
<td>4.2</td>
<td>9.5</td>
<td>3.9</td>
<td>8.5</td>
<td>3.1</td>
</tr>
<tr>
<td>4.3</td>
<td>10.1</td>
<td>3.9</td>
<td>9.2</td>
<td>3.4</td>
</tr>
<tr>
<td>4.4</td>
<td>14.0</td>
<td>4.4</td>
<td>13.2</td>
<td>3.6</td>
</tr>
</tbody>
</table>

Figure 6.15: Tracking test 4.4 with 512 \textit{samples} long window.

Some curved trajectories have been also tracked: an example can be seen in figure 6.18.
Figure 6.16: Test 4.4 corresponding error vs. window progressive number, 512 samples long window.

Figure 6.17: Effect on tracking estimation Mean Square Error of error in starting point estimation.
Figure 6.18: Example of tracking of a curved trajectory on MDF.
Chapter 7

Conclusions and future developments

The diffusion of electronic equipments in everyday life requires the development of new tangible interfaces to operate them. These interfaces have to be both cheap and resistant to hard environmental conditions. For these reasons, novel approaches are studied. The technique developed in this project work can be considered a solution to the localization and tracking problems: the two main tasks required when creating a tangible interface. After a deep study of the dispersion inside various materials, we have presented the basic ideas of the RDOR technique combined with a series of optimizations to make it feasible for real time applications. The experiments that we have performed demonstrate the good accuracy of the proposed approach, both in localization and tracking problems. We have then proposed a complete, classic HCI, but the technique can be used, for example, to develop new kinds of electronic musical instruments.

Further researches can be done in the direction of optimizing the present system by, for example, introducing variable increments to create filter banks for tracking, based on an estimation of the speed of the source. In fact, if the source moves faster, the range of distances to be searched with the dispersion compensation filters is larger; on the other hand, if the source moves slower, the range is smaller. With the knowledge about the position of the source in the previous windows we can compute an average speed, and choose the range
of distances in accordance.

Another idea can be the dynamic selection of the reference sensor by choosing the one closer to the source, in order to have the less dispersed signal as the reference.

The problem of tracking on very smooth surfaces (like plexiglass) is still open, and no tests have been run on metals. Anyway, as long as a signal is detectable, we expect the technique to work with any kind of material.

It would be also interesting to integrate a system for calculation and subtraction of edge’s reflections in order to make the system more precise and stable.

Another important part, that we didn’t deepen, is the development of a calibration system (based on the guideline presented in section 4.2) to obtain elastic properties for any kind of material in order to be able to use the system on any surface.

In this thesis we supposed that the plates are isotropic: for this reason, the dispersion depends only on the distance traveled by the wave and not on the position of the source. Further studies could also consider the effective anisotropy of the media.

A more powerful system for pattern recognition (e.g. distinguish between a touch with the nail or with the fleshy part of the fingertip) would be also very important in order to completely exploit the potentials of the HCI developed in this work.
Bibliography


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Appendices
Appendix A

Wave propagation in thin plates

This appendix is a summary of [26].

A.1 The concept of Lamb waves: structure and properties

Lamb waves refer to elastic perturbations propagating in a solid plate (or layer) with free boundaries, for which the displacements occur both in the direction of wave propagation and perpendicularly to the plane of the plate. Lamb waves represent one of the types of normal or plate modes in an elastic waveguide, in this case a plate with free boundaries. For this reason, Lamb waves are sometimes simply called normal modes in a plate. But this definition is rather loose, because another type of normal mode can exist in a plate with free boundaries, namely transverse normal modes, wherein the motion is perpendicular to the direction of propagation and parallel to the boundaries of the plate.

Consider a plane harmonic Lamb wave propagating in a plate of thickness $t = 2d$ in the positive $x$ direction (Fig. A.1). For the region occupied by the plate, the scalar potential $\phi$ and the vector potential $\psi$ of the displacements are introduced, so that the particle displacement vector $\vec{v}$ can be written in
the form:

\[ \vec{v} = \nabla \Phi + \nabla \wedge \Psi \]  \hspace{1cm} (A.1)

The wave is plane, the motion does not depend on the coordinate \( y \) and so only the component of the vector potential along the \( y \)-axis has a nonzero magnitude; we denote this component by \( \psi \). The potentials \( \phi \) and \( \psi \) are called the potentials of longitudinal (P-wave) and shear waves (S-wave), respectively, and satisfy the following wave equations:

\[ \frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial z^2} + k_\alpha^2 \Phi = 0 \]  \hspace{1cm} (A.2)

\[ \frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial z^2} + k_\beta^2 \Psi = 0 \]

with:

\[ k_\alpha = \frac{\omega}{v_\alpha} \]

\[ k_\beta = \frac{\omega}{v_\beta} \]  \hspace{1cm} (A.3)

\( k_\alpha \) and \( k_\beta \) are the P and S wave numbers, \( v_\alpha \) and \( v_\beta \) are the corresponding velocities, while \( \omega \) is the circular frequency.

The components \( u \) and \( w \) of the particle displacement along \( x \) and \( z \) axes and the stress components \( \sigma_{xx} \), \( \sigma_{zz} \) and \( \sigma_{xz} \) may be represented in terms of \( \phi \) and \( \psi \):

\[ u = \frac{\partial \Phi}{\partial x} - \frac{\partial \Psi}{\partial z} \]

\[ w = \frac{\partial \Phi}{\partial z} + \frac{\partial \Psi}{\partial x} \]  \hspace{1cm} (A.4)

\[ \sigma_{xx} = \lambda \left( \frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial z^2} \right) + 2\mu \left( \frac{\partial^2 \Phi}{\partial z^2} + \frac{\partial^2 \Psi}{\partial x \partial z} \right) \]

\[ \sigma_{zz} = \lambda \left( \frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial z^2} \right) + 2\mu \left( \frac{\partial^2 \Phi}{\partial z^2} + \frac{\partial^2 \Psi}{\partial x \partial z} \right) \]

\[ \sigma_{xz} = \mu \left( 2\frac{\partial^2 \Phi}{\partial x \partial z} + \frac{\partial^2 \Psi}{\partial z^2} - \frac{\partial^2 \Psi}{\partial x^2} \right) \]  \hspace{1cm} (A.5)

\( \lambda \) and \( \mu \) are the Lamé constants. \( \phi \) and \( \psi \) can be represented in the following
\[ \Phi = A_S \cosh(qz)e^{j(kx-\omega t)} + B_A \sinh(qz)e^{j(kx-\omega t)} \]
\[ \Psi = D_S \cosh(sz)e^{j(kx-\omega t)} + C_A \sinh(sz)e^{j(kx-\omega t)} \]  
(A.6)

with:
\[ q^2 = k^2 - k_0^2 \alpha \]
\[ s^2 = k^2 - k_0^2 \beta \]  
(A.7)

and where \( A_S, B_A, D_S \) and \( C_A \) are arbitrary constants and \( k \) is the Lamb wave number.

Lamb waves refer to elastic perturbations propagating in a solid plate with free boundaries:
\[ \sigma_{xx}|_{z=\pm d} = 0 \]
\[ \sigma_{zz}|_{z=\pm d} = 0 \]  
(A.8)

Substituting (A.6) into (A.5) and equating the indicated stresses to zero (A.8), the following system of linear homogeneous equations for the amplitudes \( A_S, B_A, D_S \) and \( C_A \) is obtained:
\[
\begin{cases}
(k^2 + s^2)A_S \cosh(qd) + (k^2 + s^2)B_A \sinh(qd) + \\ 
\quad \cdots + 2iksC_A \sinh(sd) + 2iksD_S \cosh(sd) = 0 \\
(k^2 + s^2)A_S \cosh(qd) - (k^2 + s^2)B_A \sinh(qd) + \\ 
\quad \cdots - 2iksC_A \sinh(sd) + 2iksD_S \cosh(sd) = 0 \\
2ikqA_S \sinh(qd) + 2ikqB_A \cosh(qd) + \\
\quad \cdots - (k^2 + s^2)C_A \cosh(sd) - (k^2 + s^2)D_S \sinh(sd) = 0 \\
\quad -2ikqA_S \sinh(qd) + 2ikqB_A \cosh(qd) + \\
\quad \cdots - (k^2 + s^2)C_A \cosh(sd) + (k^2 + s^2)D_S \sinh(sd) = 0
\end{cases}
\]  
(A.9)

The above system is satisfied if the following two sub-systems are satisfied:
\[
\begin{cases}
(k^2 + s^2)A_S \cosh(qd) + 2iksD_S \cosh(sd) = 0 \\
2ikqA_S \sinh(qd) - (k^2 + s^2)D_S \sinh(sd) = 0 
\end{cases}
\]  
(A.10)

\[
\begin{cases}
(k^2 + s^2)B_A \sinh(qd) + 2iksC_A \sinh(sd) = 0 \\
2ikqB_A \cosh(qd) - (k^2 + s^2)C_A \cosh(sd) = 0 
\end{cases}
\]  
(A.11)

The subsystems have nontrivial solutions only when their determinants are equal to zero. This leads to two characteristic equations, determining the
eigenvalues of the wave number $k$:

\[(k^2 + s^2) \cosh(qd) \sinh(sd) - 4qsk^2 \sinh(qd) \cosh(sd) = 0 \quad (A.12)\]

\[(k^2 + s^2) \sinh(qd) \cosh(sd) - 4qsk^2 \cosh(qd) \sinh(sd) = 0 \quad (A.13)\]

With these equations, from the subsystem A.10 an expression for $D_S$ in terms of $A_S$ and from the subsystem A.11 an expression for $C_A$ in terms of $B_A$ can be obtained. Inserting them into A.6 we arrive at the following relations for the desired potentials:

\[
\begin{align*}
\Phi &= A_S \cosh(q_s z) e^{j(k_s x - \omega t)} + B_A \sinh(q_a z) e^{j(k_a x - \omega t)} \\
\Psi &= \frac{2ik_s q_s \sinh(q_s d)}{(k_s^2 + s^2) \sinh(s_s d)} A_S \sinh(s_s z) e^{j(k_s x - \omega t)} + \frac{2ik_a q_a \cosh(q_a d)}{(k_a^2 + s^2_a) \cosh(s_a d)} B_A \cosh(s_a z) e^{j(k_a x - \omega t)}
\end{align*}
\]

(A.14)

Here, $k_s$ are the values of $k$ satisfying the equation A.12 and $k_a$ are the values satisfying the A.13:

\[
\begin{align*}
q^2_{s,a} &= k^2_{s,a} - k^2 \\
s^2_{s,a} &= k^2_{s,a} - k^2
\end{align*}
\]

(A.15)

The displacement components $u$ and $w$ may be calculated from the equation A.14 and the relations A.4:

\[
\begin{align*}
u &= u_s + u_a \\
w &= w_s + w_a
\end{align*}
\]

(A.16)

where:

\[
\begin{align*}
u_s &= A k_s \left( \frac{\cosh(q_s z)}{\sinh(q_s d)} - \frac{2q_s s_s \cosh(s_s z)}{k^2_s + s^2 \sinh(s_s d)} \right) e^{j(k_s x - \omega t - \frac{\pi}{2})} \\
w_s &= -A q_s \left( \frac{\sinh(q_s z)}{\sinh(q_s d)} - \frac{2ik_s^2 \sinh(s_s z)}{k^2_s + s^2 \sinh(s_s d)} \right) e^{j(k_s x - \omega t)}
\end{align*}
\]

(A.17)

\[
\begin{align*}
u_a &= B k_a \left( \frac{\sinh(q_a z)}{\cosh(q_a d)} - \frac{2q_a s_a \sinh(s_a z)}{k^2_a + s^2 \cosh(s_a d)} \right) e^{j(k_a x - \omega t - \frac{\pi}{2})} \\
w_a &= -B q_a \left( \frac{\cosh(q_a z)}{\cosh(q_a d)} - \frac{2ik_a^2 \cosh(s_a z)}{k^2_a + s^2 \cosh(s_a d)} \right) e^{j(k_a x - \omega t)}
\end{align*}
\]

(A.18)

Here $A$ and $B$ are arbitrary constants.

The expressions A.14-A.18 and the equations A.12 and A.13 describe two groups of waves, each of which satisfies the wave equations of motion and boundary conditions (both can propagate in the plate independently of one another). Analyzing the expressions A.17 and A.18 it can be see right away
that the first group of waves, indicated by the subscript $s$, describes waves in which the motion occurs symmetrically with respect to the plane $z = 0$. The second group, indicated by the subscript $a$, describes waves in which the motion is antisymmetrical with respect to $z = 0$. The waves of the first group are called symmetrical Lamb waves, those of the second group are called antisymmetrical Lamb waves. The deformation of the plate in the $z$ direction during the propagation is illustrated in Fig. A.2.

A.2 The number of Lamb waves and critical frequencies

In a plate of thickness $2d$ at a frequency $\omega$ there can exist a finite number of symmetrical and antisymmetrical Lamb waves, differing from one another by their phase, group velocities, distribution of the displacements and stresses throughout the thickness of the plate. The number of symmetrical waves is determined by the number of real roots of equation A.12, while the number of antisymmetrical waves by the roots of the equation A.13. Every root defines a wave number $k_{s,a}$ or phase velocity $v_{s,a}$ of the corresponding wave. It can be shown that, in addition to the finite number of real roots $k_{s,a}$, both of the indicated equations have for any $\omega$ and $d$ an infinite set of purely imaginary roots, corresponding to in-phase motions of the plate which decay exponentially along the $x$-axis. Since the interest here is on propagating waves, only the real roots of the characteristic equations A.12 and A.13 will be considered.

For $\omega d \to 0$ the previous equations have only one root each. The root of
the A.12 corresponds to the so called zeroth symmetrical normal mode, which will be designate \( s_0 \), while the root of the A.13 represents the zeroth antisymmetrical mode \( a_0 \). As \( \omega d \) increases, the roots \( k_{s,0} \) and \( k_{a,0} \) vary in magnitude and for definite ratios between \( \omega \) and \( d \) new roots appear, corresponding to the first, second and higher symmetrical \((s_1, s_2, \ldots, s_n)\) and antisymmetrical \((a_1, a_2, \ldots, a_n)\) Lamb waves. The values of \( \omega \) and \( d \) at which new roots appear are called the critical thicknesses and frequencies. The relations between the critical thicknesses and transverse and longitudinal wavelengths are for the symmetrical modes:

\[
\begin{align*}
2d &= \frac{\lambda_p}{2}, \frac{3\lambda_p}{2}, \frac{5\lambda_p}{2}, \ldots \\
2d &= \lambda_s, 2\lambda_s, 3\lambda_s, \ldots
\end{align*}
\]

(A.19)

and for antisymmetrical modes:

\[
\begin{align*}
2d &= \lambda_p, 2\lambda_p, 3\lambda_p, \ldots \\
2d &= \frac{\lambda_p}{2}, \frac{3\lambda_p}{2}, \frac{5\lambda_p}{2}, \ldots
\end{align*}
\]

(A.20)

At critical frequencies the wave numbers \( k_{s,a} \to 0 \) and the new symmetrical or antisymmetrical mode represents a standing longitudinal or transverse wave in the plate.

The total number of symmetrical modes \( N_s \) that are possible in a plate of given thickness \( 2d \) and at the frequency \( \omega \) is equal to:

\[
N_s = 1 + \left\lfloor \frac{2d}{\lambda_s} \right\rfloor + \left\lfloor \frac{2d}{\lambda_p} + \frac{1}{2} \right\rfloor
\]

(A.21)

The total number of antisymmetrical modes is:

\[
N_a = 1 + \left\lfloor \frac{2d}{\lambda_p} \right\rfloor + \left\lfloor \frac{2d}{\lambda_s} + \frac{1}{2} \right\rfloor
\]

(A.22)

Note that the brackets in this case indicate the nearest integer part of the number that they enclose.

A.3 Phase velocities of Lamb waves

The phase velocity \( v \) is the fundamental characteristic of the Lamb wave and once it is known, the wave number can be determined and the stresses and
Figure A.3: Phase velocities of the modes $s_0$, $s_1$, $s_2$, $a_0$, $a_1$, $a_2$, corresponding to a Poisson ratio $\nu = 0.34$.

displacements at any point of the plate calculated. The phase velocity is found by numerical solution of the characteristic equations A.12 and A.13, which are suitably rewritten in the following dimensionless form. For the symmetrical modes:

$$\frac{\tan(\bar{d}) \sqrt{(1 - \xi_s^2)}}{\tan(d) \sqrt{(\xi^2 - \xi_s^2)}} + \frac{4\xi_s^2 \sqrt{(1 - \xi_s^2)} \sqrt{(\xi^2 - \xi_s^2)}}{(2\xi_s^2 - 1)^2} = 0 \quad (A.23)$$

and for antisymmetrical modes:

$$\frac{\tan(\bar{d}) \sqrt{(1 - \xi_a^2)}}{\tan(d) \sqrt{(\xi^2 - \xi_a^2)}} + \frac{4\xi_a^2 \sqrt{(1 - \xi_a^2)} \sqrt{(\xi^2 - \xi_a^2)}}{(2\xi_a^2 - 1)^2} = 0 \quad (A.24)$$

where:

$$\bar{d} = k_s d \quad (A.25)$$

$$\xi_{s,a}^2 = \frac{v_{s,a}^2}{v^2} \quad (A.26)$$

$$\xi^2 = \frac{v_\beta^2}{v_\alpha^2} \quad (A.27)$$

Many authors have performed calculations of the phase velocities and their dependence on the plate thickness and frequency (dispersion curves). In Fig. A.3 the phase velocities of the modes $s_0$, $s_1$, $s_2$, $a_0$, $a_1$, $a_2$, corresponding to a Poisson ratio $\nu = 0.34$ can be seen.
A.4 The motion in Lamb waves

Knowing the dispersion curves for the phase velocities, the motion in waves of various order numbers can be discussed. As a characteristic of the motion it is sufficient to know the displacements along the $x$ and the $z$ axes at different points of the plate. The displacements can be used to calculate the stresses in the wave, making use of the following relations:

\[
\sigma_{xx} = \lambda \left( \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} \right) + 2\mu \left( \frac{\partial u}{\partial x} \right)
\]

\[
\sigma_{xz} = \mu \left( \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right)
\]

\[
\sigma_{xx} = \lambda \left( \frac{\partial w}{\partial z} \right) + 2\mu \left( \frac{\partial w}{\partial z} \right)
\]

The modes $a_0$ and $s_0$ differ qualitatively from all the others in their velocity dispersion curves and in that they exist for any frequencies and plate thicknesses. For small plate thicknesses, when $k_{s0,a0}d \to 0$ these modes represent a longitudinal ($s_0$) and flexural ($a_0$) wave in a thin plate. In this case their displacements are described by the following expressions, derived as the result of limiting transitions from equations A.17 and A.18:

\[
u_{s0} = Ak_\beta \frac{k^2 - s^2}{k^2 + s^2 + k^2d} \sin(k_s x - \omega t)
\]

\[
w_{s0} = Ak_\beta \frac{k^2 - s^2}{k^2 + s^2 + k^2d} \cos(k_s x - \omega t)
\]

\[
u_{a0} = Bk_\beta \frac{k^2}{2a} \sin(k_a x - \omega t)
\]

\[
w_{a0} = Bk_\beta \frac{k^2}{2a} \cos(k_a x - \omega t)
\]

It's apparent from the equations that displacements along the $x$ axis prevail in the longitudinal wave, the displacement amplitude being the same at all points of the plate. The displacement in the transverse direction due to the Poisson effect is less than the longitudinal displacement by a factor of approximately $1/(k_\beta d)$. It is a maximum on the surface and equal to zero in the median plane of the plate. In the flexural wave, on the other hand, the transverse displacement $w$ prevails, its amplitude being the same at all points of the plate. The longitudinal displacement is equal to zero in the median plane and is a maximum on the surface of the plate; on the average it is less than the transverse displacement by a factor of $1/(k_\beta d)$.

As the thickness of the plate increases the properties of the waves $a_0$ and
$s_0$ change; they become more and more "like" one another. For $k_\beta d >> 1$ their phase velocities tend to the Rayleigh wave phase velocity $v_R$ and the displacements become localized near the free boundaries of the plate.
Appendix B

Tarantola’s inversion theory

This appendix is a summary of [26].

B.1 Inverse problem: Tarantola’s approach

The scientific procedure to study a physical system can be divided into three steps:

1. Parameterization of the system: discovery of a minimal set of model parameters whose values completely characterize the system.

2. Forward modeling: discovery of the physical laws allowing, for given values of the model parameters, to make predictions on the results of measurements on some observable parameters, also called data.

3. Inverse modeling: use of the actual results of some measurements parameters to infer the actual values of the model parameters.

The model can be achieved from the data (inversion or inverse problem) by using the Tarantola’s approach: Central in this method is the concept of state of information over a parameter set. Its postulated that the most general way of describing such a state of information is by defining a probability density over the parameter space. It follows that the result of the measurements of the data, the a priori information on model parameters and the information on the physical correlations between observable parameters and model parameters can, all of them, be described using probability densities. The general inverse
problem can be set as a problem of combination of all this information (A. Tarantola).

B.2 Model space and data space

A physical system can be completely characterized by a set of parameters \( m = \{m_1, m_2, m_3, \ldots, m_M\} \) called model. To obtain information on the model parameters, some observations during a physical experiment has to be performed, i.e., a measurement of a set of observable parameters \( d = \{d_1, d_2, d_3, \ldots, d_N\} \) called data has to be performed.

The choice of the model parameters is generally not unique and each particular choice is a parameterization of the physical system. Independently of any particular parameterization it is possible to introduce a manifold, an abstract space of points where each point represents a conceivable model of the system. This manifold is named the model space and it is represented by the symbol \( \mathcal{M} \).

It is also possible to define the data space \( \mathcal{D} \), whose points correspond to conceivable results of the measurements. \( M \) and \( N \) are called the cardinality of the model space and of the data space.

The relation between data and model can be written in the form:

\[
\mathbf{d} = g(\mathbf{m})
\]

(B.1)

where \( g \) is the function (usually nonlinear) describing the data for a given values of the model parameters (forward operator).

B.3 Prior information and measurements uncertainties

The information which is obtained independently of the results of measurements is called prior (or apriori) information; the probability density representing this information is assumed to be Gaussian, it is defined over the
model space $\mathcal{M}$ and denoted by:

$$
\rho_M(m) = ((2\pi)^M \det(C_M))^{-\frac{1}{2}} \exp\left[\frac{1}{2}(m - m_{pr})^T C_M^{-1}(m - m_{pr})\right] \quad (B.2)
$$

where $m_{pr}$ is the prior model and $C_M$ the corresponding covariance matrix.

All physical measurements are subjected to uncertainties. Therefore the result of a measurement act is not simply an "observed value", but a "state of information" acquired on some observable parameters and can be represented by a probability density assumed to be Gaussian and defined over $\mathcal{D}$:

$$
\rho_D(d) = \mu_D d((2\pi)^N \det(C_D))^{-\frac{1}{2}} \exp\left[\frac{1}{2}(d - d_{obs})^T C_D^{-1}(d - d_{obs})\right] \quad (B.3)
$$

where $d_{obs}$ and $C_D$ are the observed data and the corresponding covariance matrix.

By definition, the apriori information on model parameters and the observations are independent; it is thus possible to define a joint prior information, defined over the space $\mathcal{D} \times \mathcal{M}$, by the probability density:

$$
\rho(d, m) = \rho_D(d)\rho_M(m) \quad (B.4)
$$

If the homogeneous probability distribution is defined as:

$$
\mu(x) = \lim_{\text{dispersion} \to \infty} \rho(x) \quad (B.5)
$$

then:

$$
\mu(d, m) = \mu_D(d)\mu_M(m) \quad (B.6)
$$

### B.4 Theoretical information

The information on the physical correlations between observable parameters and model parameters is called theoretical probability density, defined over the space $\mathcal{D} \times \mathcal{M}$ as:

$$
\Theta(d, m) = \Theta(d|m)\mu_M(m) \quad (B.7)
$$
and the modelling errors can be assumed to be Gaussian and described by:

\[ \Theta(d|m) = ((2\pi)^N \det(C_T))^{-\frac{1}{2}} \exp\left[-\frac{1}{2}(d - g(m))^T C_T^{-1}(m - g(m))\right] \] (B.8)

This probability density is defined over the space \( D \) and \( C_T \) is the theoretical covariance matrix.

### B.5 The solution to the inverse problem

It has been seen that the prior probability density \( \rho(d,m) \) represents both information obtained on the observable parameters (data) \( d \), and apriori information on model parameters (model) \( m \). It has been also seen that the theoretical probability density \( \Theta(d,m) \) represents the information on the physical correlations between \( d \) and \( m \), for instance as obtained from a physical law.

The conjunction of these two states of information gives the posterior (or aposteriori) state of information of the system (Tarantola's postulate):

\[ \sigma(d,m) = k \rho(d,m) \Theta(d,m) \]  

\[ \mu(d,m) (B.9) \]

where \( \mu(d,m) \) represents the homogeneous state of information and where \( k \) is a normalization constant.

Once the posterior information in the \( D \times M \) space has been defined, the a posterior information in the model space (describing the solution of the inversion) is given by the marginal probability density:

\[ \sigma_M(m) = \int_D \sigma(d,m) dd \]  

(B.10)

Substituting the equation B.9 in the integral B.10, considering the independence assumptions B.4 and B.5 and the definition B.8, the following relations is obtained:

\[ \sigma_M(m) = k \int_D \frac{\rho_M(m) \rho_D(d) \Theta(d|m) \mu_M(m) \mu_D(d)}{\mu_M(m) \mu_D(d)} dd d = k \rho_M(m) \int_D \frac{\rho_D(d) \Theta(d|m) dd}{\mu_D(d)} \]  

(B.11)

Assuming the Gaussian probability densities of the equations B.2, B.3 and
B.8, the previous relation leads to:

\[
\sigma_M(m) = ke^{-\frac{1}{2}[g(m) - d_{obs})^T C_d^{-1}(g(m) - d_{obs}) + (m - m_{pr})^T C_M^{-1}(m - m_{pr})]} \tag{B.12}
\]

with \( C_d = C_D + C_T \) the covariance matrix that considers the uncertainties due to both the modelling and the measurements.

The solution of the inversion can be achieved maximizing the posterior probability density of the model or minimizing \( S(m) \), the argument of the exponential expressed by the equation B.12, changed of the sign:

\[
S(m) = -\frac{1}{2} [(g(m) - d_{obs})^T C_d^{-1}(g(m) - d_{obs}) + (m - m_{pr})^T C_M^{-1}(m - m_{pr})] \tag{B.13}
\]

The minimization can be achieved calculating \( S'(m) \) and imposing \( S'(m) = 0 \) can be expressed by:

\[
S(m) = \frac{1}{2} \left\{ \sum_i \sum_j [(g_i(m) - d_i)[C_d^{-1}]_{ij}(g_j(m) - d_j)] + \ldots \\
\ldots + \sum_\alpha \sum_\beta [(m_\alpha - m_{pr,\alpha})[C_M^{-1}]_{\alpha\beta}(m_\beta - m_{pr,\beta})] \right\} \tag{B.14}
\]

The derivative of \( S(m) \) with respect to the component of the model \( m_\alpha \) is:

\[
\frac{\partial S(m)}{\partial m_\alpha} = \left\{ \sum_i \sum_j [G_{i\alpha}[C_d^{-1}]_{ij}(g_j(m) - d_j)] + \sum_\beta [(C_M^{-1})_{\alpha\beta}(m_\beta - m_{pr,\beta})] \right\} \tag{B.15}
\]

where \( G_{i\alpha} \) is the Jacobian matrix, whose components are:

\[
G_{i\alpha} = \frac{\partial g_i}{\partial m_\alpha} \tag{B.16}
\]

Therefore the derivative of \( S(m) \) with respect to the model \( m \) is equal to:

\[
\frac{\partial S(m)}{\partial m} = G^T C_d^{-1}(g(m) - d) + C_M^{-1}(m - m_{pr}) \tag{B.17}
\]
Imposing $S'(\bar{m}) = 0$ we obtain:

$$\frac{\partial S(m)}{\partial m} = 0 \Rightarrow C_M^{-1}(m - m_{pr}) = -G^T C_d^{-1}(g(m) - d) \quad (B.18)$$

We can add to both of the terms of the previous equation the quantity $G^T C_d^{-1}G(m - m_{pr})$, deriving the expression:

$$G^T C_d^{-1}G(m - m_{pr}) + C_M^{-1}(m - m_{pr}) = G^T C_d^{-1}G(m - m_{pr}) - G^T C_d^{-1}(g(m) - d) \quad (B.19)$$

that leads to:

$$\bar{m} = m_{pr} - \left[ G^T C_d^{-1}G + C_M^{-1} \right]^{-1} G^T C_d^{-1} \left[ (g(m) - d) - G(m - m_{pr}) \right] \quad (B.20)$$

which is an implicit equation (the unknown $\bar{m}$ is present both on the right and on the left of the previous equation) and can be solved implementing the following iterative fix point algorithm:

$$m_{k+1} = m_{pr} - \left[ G_k^T C_d^{-1}G_k + C_M^{-1} \right]^{-1} G_k^T C_d^{-1} \left[ (g(m_k) - d_{obs}) - G_k(m_k - m_{pr}) \right] \quad (B.21)$$

where $d_{obs}$ is the vector of the observed data, $m_k$ and $m_{k+1}$ are the vectors of the model at the $k$ and $k + 1$ iterations and $G_k$ is the Jacobian matrix, linearized around the model $m_k$. The iterative algorithm stops when:

$$|m_{i,k+1} - m_{i,k}| < \varepsilon \quad \forall \quad i = 1, \ldots, M \quad (B.22)$$

The accuracy of the solution can be described by the posterior model covariance matrix:

$$\hat{C}_M = (G_k^T C_d^{-1}G_k + C_M^{-1})^{-1} \quad (B.23)$$
Appendix C

Tracking tests figures

In this appendix are collected all the figures from tracking tests that are not shown in section 6.2.

Figure C.1: Tracking test 2.2: estimated and theoretic trajectories inserted in the panel’s usable area.

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Figure C.2: Test 2.2 corresponding error vs. window progressive number.

Figure C.3: Tracking test 2.3: estimated and theoretic trajectories inserted in the panel’s usable area.
Figure C.4: Test 2.3 corresponding error vs. window progressive number.

Figure C.5: Tracking test 3.1: estimated and theoretic trajectories inserted in the panel’s usable area.
Figure C.6: Test 3.1 corresponding error vs. window progressive number.

Figure C.7: Tracking test 3.2: estimated and theoretic trajectories inserted in the panel’s usable area.
Figure C.8: Test 3.2 corresponding error vs. window progressive number.

Figure C.9: Tracking test 4.2: estimated and theoric trajectories inserted in the panel’s usable area.
Figure C.10: Test 4.2 corresponding error vs. window progressive number.

Figure C.11: Tracking test 4.3: estimated and theoretic trajectories inserted in the panel's usable area.
Figure C.12: Test 4.3 corresponding error vs. window progressive number.

Figure C.13: Tracking test 4.4: estimated and theoretic trajectories inserted in the panel’s usable area.
Figure C.14: Test 4.4 corresponding error vs. window progressive number.
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